

# The Future of Statistics in our Schools and Colleges

Roger Porkess



**ROYAL  
STATISTICAL  
SOCIETY**

**The Actuarial Profession**  
making financial sense of the future



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## About the Royal Statistical Society

The Royal Statistical Society is a learned society for statistics and a professional body for statisticians. It was founded in 1834 as the Statistical Society of London (LSS) and became the RSS (Royal Statistical Society) by Royal Charter in 1887. In 1993 the RSS merged with the Institute of Statisticians. Today the society has more than 7000 members around the world, of whom some 1500 are professionally qualified with the status of Chartered Statistician.

The Society is active in a wide range of activities, reflecting the diversity within the discipline of statistics, and it is one of the world's leading statistical societies. The RSS focuses strongly on statistical education in its broadest sense, as stated in its Royal Charter, and offers a range of educational qualifications and continuing professional development opportunities for statisticians.

In 2010 the Society launched the *getstats* campaign. With support from the Nuffield Foundation, the campaign aims to increase statistical literacy and wants to raise the profile of statistics and its increasing relevance in today's data-rich society.

## About the Actuarial Profession

The Actuarial Profession represents and regulates the members of the Institute and Faculty of Actuaries, the chartered professional body for actuaries in the UK.

Actuaries' training is founded on mathematical and statistical techniques used in insurance, pension fund management and investment and then builds the management skills associated with the application of these techniques.

Actuaries work in insurance, pensions, healthcare, investment and banking as well as in the management of risk, and so are directly involved in the provision of different sorts of financial products across a range of market segments.

As part of the Profession's commitment to supporting education, we sponsor the annual UK Maths Trust Maths Challenges in which 250,000 pupils from schools and colleges take part and the annual Enterprising Maths Challenge in Scotland which attracts 60,000 participants.

## Foreword – Jane Curtis President of the Institute and Faculty of Actuaries

The use and interpretation of statistics are critical in today's society. From working out the best deals in the supermarket to understanding trends and probabilities that affect decisions in business and politics, people's ability to interpret data and their sources has never been more important.



Throughout their lives, individuals also have to make financial decisions ranging from the simple to the complex. A knowledge of the theory and practical use of statistics can only help make these decisions more informed.

If the purpose of education is to help pupils and students understand and make sense of the world around them then I believe there is no better way of ensuring this than by giving them a firm grounding in statistics.

Throughout my career as an actuary, I have taken a keen interest in the education of our student members and have seen, first hand, how they are able to apply their mathematical and statistical knowledge to provide solutions to financial and business problems.

Statistics is also central to the day to day work of actuaries and we appreciate how essential gaining a good grounding in and knowledge of the application and use of statistics in schools and colleges are. This is why the Actuarial Profession is pleased to be sponsoring this timely and relevant report.

I would like to thank Roger Porkess for his hard work in producing such an interesting and comprehensive report that I am sure will make a very useful contribution to the debate.

## Foreword – Valerie Isham President of the Royal Statistical Society

Since its creation over 175 years ago the Royal Statistical Society has promoted statistics as one of the essential life skills which every citizen should have. Indeed, one of the five objectives in the Society's Charter is 'To promote the public understanding of statistics and the competent use and interpretation of statistics'.

Although this public understanding is being promoted in different ways and at different life stages it is clear it needs to be anchored in a sound and relevant formal education from an early age. It is therefore one of the Society's key aims to promote the competent teaching of statistics and interpretation of data in all schools, colleges and higher education.



However, it is not always clear precisely how the current provision of education in our primary and secondary schools handles the subject matter of statistics. Statistics is a multifaceted discipline, and although it clearly has strong roots in mathematics it is also very important in its application in a wide range of fields, from science and engineering to the financial sector, the social sciences and sports. In addition, in today's complex world, it is increasingly important for every citizen to be able to make sense of key statistical concepts, such as risk and probability, on an almost daily basis.

I am therefore delighted that, with support from the Actuarial Profession, we have been able to commission this very timely report by Roger Porkess. As an eminent UK educational specialist, he is extremely well positioned to take stock of the current provision of statistics in primary and secondary education in England. The results of his research clearly highlight the issues and problems which hamper the relevant provision of sound statistical knowledge. They clearly support the argument that statistics cannot be ignored in either mathematics, or in a wide range of other taught subjects where problem definition, data collection, analysis and interpretation skills are essential to strengthen subject-specific knowledge.

The recommendations flowing from this research are highly relevant not only for education policy development, but for policy development in other areas such as the labour market, industry and science. The Society will strongly pursue the recommendations and promote their relevance to all policy makers to ensure statistics is recognised as an essential discipline in all facets of the education landscape.





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## Executive summary and recommendations

The report provides a summary of the statistics currently learnt in schools and colleges in England; evidence is also drawn from Wales and Northern Ireland and so the report is relevant to those parts of the United Kingdom. Based on this evidence, a number of recommendations are made for the improvement of this provision.

### Statistics in our national life

Statistics is about using data as the evidence on which to make decisions and to solve problems. It has widespread applications in policy, society, the economy and the academic world.

**Recommendation 1:** The increasing importance of statistics to our national life should be recognised in our evolving education system.

Digital technology is providing data on a scale that was unimaginable just a few years ago, and this trend is set to continue. Our national prosperity is closely linked to our ability to control, understand and make use of this supply of data.

**Recommendation 2:** Policy makers need to appreciate that the need for statistics is not going to go away; instead it will increase as ever more data become available. They need to acknowledge the central role that statistics plays in the current and future economy, and its importance for decision making.

There is now a widespread recognition that our provision of mathematics and statistics has not kept abreast of changing needs, and is now inadequate. Changes are expected in the coming years and these provide opportunities for a better provision of statistics for all students.

**Recommendation 3:** National education policy should ensure that all students are equipped with a working knowledge of basic statistics, including the necessary associated mathematical competence, and an appreciation of how it impacts on their daily lives.

It is particularly important that those responsible for education policy at school and college level recognise the importance of statistics in most degree courses, in employment as well as in enabling people to be informed citizens who evaluate evidence when making decisions.

**Recommendation 4:** Those responsible for deciding what courses are made available in schools and colleges, and for advising students about which of them to follow, need to understand how important statistics is likely to be for students in the next stages of their lives.

## Statistics in the school curriculum

Statistics is an inter-disciplinary subject. However during the years of compulsory education, up to the age of 16, it is taught as part of the mathematics curriculum, with students also using particular techniques in several other subjects. A similar pattern pertains post-16 but with the difference that mathematics, being no longer compulsory, is taken by only a small minority of each cohort. Consequently most students currently learn no statistics between the ages of 16 and 18.

Recommendation 5: School and college mathematics departments should ensure they have the expertise to be the authorities on statistics within their institutions. Mathematics departments should be centres of excellence for statistics, providing guidance on correct usage and good practice.

There are clear advantages in statistics remaining in the mathematics curriculum.

Recommendation 6: Under present conditions, statistics is best placed in the mathematics curriculum.

However, there are also dangers. Statistics could become marginalised and its distinctive nature as a discipline could be ignored or not understood. There is also the possibility that current problems in our statistics provision are not adequately addressed. To meet these concerns, it is important that statistics is appropriately represented, alongside mathematics, when policy decisions are being made. The Royal Statistical Society is well placed to take a lead in this.

Recommendation 7: To ensure that sufficient account is taken of the importance of statistics, and of its special requirements, it should be represented separately from mathematics, but alongside it, when policy decisions are being made.

At present there is a lack of coordination between the statistics in the mathematics curriculum and that needed in other subjects. Some quite straightforward topics that are important for other subjects are not covered in mathematics. There are also topics that students may meet in mathematics but long after they have met them elsewhere.

Recommendation 8: The curriculum should be designed so that, wherever possible, students have met statistical techniques in mathematics before they need to use them in other subjects.

A particularly important mismatch occurs with hypothesis testing which is required in a number of subjects at AS and A Level. It is taught in mathematics but in most syllabuses students either do not reach it, or do so long after they have used it in their other A levels.

Recommendation 9: The first statistics course in AS and A level Mathematics, usually called Statistics 1, should contain hypothesis testing to support students using it in other subjects.

## Assessment of statistics

Statistics is a practical subject. Its importance derives from its use in problem solving, providing the evidence on which decisions can be made, and this involves analysing problems and then collecting suitable data. However, these processes are not recognised in the formal assessment system for mathematics at any level: Key Stage 2, GCSE or A level.

**Recommendation 10:** Statistics is not adequately served by the assessment techniques used on current mathematics papers. This needs to be improved.

Assessment has a strong influence on classroom practice. A consequence of the present arrangements is that, at almost all levels, most students do not engage in processes that are intrinsic to the nature of statistics, and so learn neither how to carry them out nor how important they are.

**Recommendation 11:** The assessment techniques used should ensure that, at every level, students carry out work covering all the processes required to use statistics to solve problems and make decisions.

The report includes examples of assessment styles used in a number of other subjects. Many of these are designed to influence classroom practice, ensuring that the work students do gives them experience of the nature of statistics. Adopting such methods would require a change of culture among some mathematics examiners, moving away from answers being either right or wrong.

**Recommendation 12:** The assessment of statistics within mathematics should be informed by good practice in other subjects.

## Statistics in the National Curriculum

This report coincides with a revision to the National Curriculum for education up to the age of 16. The intention is that a new National Curriculum will provide the basis for an improved all round school education. This must, of course, foster statistics.

**Recommendation 13:** The new National Curriculum should ensure not only that students meet a suitable statistics curriculum at all ages up to 16 but also that it prepares them for a future in which many of them will be using statistics in a wide variety of contexts throughout the rest of their lives.

The new National Curriculum should support and promote the very good cross-curricular practice to be found in some primary schools. In addition, there is scope for some increase in the content expected at this level.

**Recommendation 14:** The programme of study for Key Stage 2 should include the data handling cycle, as is currently the case for Key Stages 3 and 4.

The statistics in GCSE Mathematics is repetitive and there is room for some extra content. Some topics are needed to ensure that other subjects are well supported, but there would probably still be space for some extra material.

**Recommendation 15:** The statistics content within mathematics, up to GCSE, should include some topics that are either not currently covered or are only treated lightly.

The new National Curricula in several other subjects could, with advantage, refer to data and statistics.

**Recommendation 16:** In all relevant subjects, revisions to the National Curriculum should be considered in the light of the increasing emphasis on quantitative methods.

## **Statistics for all post-16 students**

The National Curriculum covers up to age 16. Beyond that there is currently no requirement to do any mathematics or statistics. However, a number of recent reports have recommended that everyone should continue mathematics to the age of 18, bringing this country into line with the rest of the developed world, and the government has agreed that this should happen, with a ten-year timescale. This will not involve everyone doing AS or A level Mathematics but will require new courses to be developed, designed for those who currently typically give up mathematics at 16. Statistics is expected to feature prominently in them.

**Recommendation 17:** The prospect of new courses for mathematics and statistics post-16 is to be welcomed; there should be major involvement from the world of statistics in their design.

The target students for these new courses will include many who are currently glad to give up mathematics at 16 and so will be reluctant to continue for another two years. Consequently it is important that they are motivating; one requirement for this is that they are relevant to students' aspirations. For many of those going on to higher education, inference will be important, whereas those going into employment are likely to meet quality control and statistical process control. Everyone will benefit from a better understanding of risk.

**Recommendation 18:** New courses for post-16 students will require careful design. Their statistics content must be up-to-date and relevant to the future lives of the target students, whether in higher education or employment.

# 1 Rationale

## 1.1 Introduction

The aim of this report, and the underlying research, is to provide an evidence base for the further development of the statistics provision in our schools and colleges. It follows an earlier paper, *50 years of statistics teaching in English schools*<sup>1</sup>, which highlighted the considerable progress made over the period covered. Clearly there is a legacy to be built upon, and as a matter of urgency because the situation is not static.

The need for development of the statistics provision in schools and colleges is driven by changes in higher education and employment consequent upon the increasing availability of data, and the need to analyse and interpret them. Quantitative methods are now taking a more central place in many undergraduate courses and this is an international trend that universities in this country cannot avoid. In the workplace the use of statistical processes is becoming ever more linked to success in a competitive environment. However, it is not just within these confines that the changes are being felt but in society at large; at all levels, decision making is increasingly based upon statistical evidence.

The cause of these changes is not hard to find. Computers, and more generally digital technology, have made it possible to collect and process data on a scale that was previously unimaginable. The development of these technologies is not, of course, about to come to a sudden halt; year on year we can expect more data to become available. In addition, the present government is committed to making more of its data available to the public<sup>2</sup>.

It is sometimes suggested that computers remove the need for mathematics and statistics. Nothing could be further from the truth. Computers produce data; it is up to people to decide how to analyse and interpret them, and indeed what data to seek.

Although data now pervade our lives and our way of understanding the world we live in, many people are so frightened at the sight of numbers that they are unable to engage with them. This is a disaster for those individuals, and also for the nation as a whole. Increasingly we need people who are at ease with data, in their many forms, and able to interpret them. All young people should be provided with these skills during their school years, through both mathematics and particularly statistics. This report is designed to help this to happen.

However, implementing its recommendations will depend on decision makers appreciating just how important statistics is for the future of the country. Many already understand this, but some may still need to be convinced.

## 1.2 Research questions

The research began with an audit of what statistics is currently being taught to the various age groups in schools and colleges and where it is happening in the curriculum. From the outset it was the intention that this would provide an evidence base for the future development and improvement of the statistics provision in our schools and colleges. However, taking the findings forward would require a number of key issues to be addressed, leading to a framework of six questions.

### 1.2.1 What statistics is currently being learnt in our schools and colleges?

The current provision is described in Sections 5, 6 and 7 of the report, covering, respectively, primary school children, 11–16-year-olds and those post-16.

### 1.2.2 How can statistics at this level best be classified?

Determining the statistical content of a wide variety of courses produced a substantial amount of detailed information. The question of how to present it in a useful form raised deep and fundamental issues about the nature of statistics as a discipline. This is considered early in the report, in Section 4, and the answer adopted is critical to much of the subsequent analysis.

### 1.2.3 Are students given a good experience of statistics? Are they being provided with a coherent body of knowledge or is it fragmented?

These questions pervade much of the report, with the conclusions being summarised in Sections 8 and 9.

### 1.2.4 Are there missing topics?

The report looks at the statistics used across the curriculum, not just in mathematics. Implications for current syllabuses are considered in Section 9. A number of topics are identified as missing or misplaced.

### 1.2.5 Where in the curriculum would statistics best be taught?

At present statistics forms part of the mathematics curriculum. Particular techniques are also taught in many other subjects. It is sometimes argued that statistics would be better learnt in context, outside mathematics. The question of where statistics is best located is discussed in Section 8.

### 1.2.6 Where should ownership of statistics at this level lie?

Statistics is both a subject in its own right and one that is used across the whole curriculum and not just in mathematics. Leadership is needed at a strategic level. The report ends with a discussion of this issue in Section 10.



## 1.3 Other research

This report benefited from information from several other research projects which were going on at the same time. In addition, a number of particularly influential recent reports and events are described in Section 2.

### 1.3.1 ACME and Mathematical Needs

During 2010–2011, the Advisory Committee for Mathematics Education (ACME) ran a major research project into the mathematical needs of the nation. Two elements of ACME's work, those covering the workplace and higher education, were particularly relevant to the ongoing work on this statistics report.

A large number of employers and employees, across the different sectors of industry, were interviewed and their views were brought together in the final report<sup>3</sup>. The findings include the fact that the workplace today is pervaded by data which people at all levels in a company need to interpret and analyse.

The ACME research also found that a substantial majority of undergraduates are on courses that require mathematics beyond GCSE and that in many cases this is largely statistics. The report estimated that this is true of about 330,000 students per year but that the school and college system provides fewer than 125,000 with this background learning.

### 1.3.2 SCORE

The Science Community Representing Education (SCORE) is a collaboration of organisations working together on science education policy for 5–19-year-olds. The SCORE membership organisations are the Association for Science Education, Institute of Physics, Royal Society, Royal Society of Chemistry and Society of Biology.

In 2009, SCORE reviewed the 2008 GCSE Science and Additional Science examination papers; this exercise was subsequently repeated for the 2009 papers and, at the time of writing, those for 2010 are under review. This process included determining the mathematics being assessed on the papers. The report on the 2008 examinations<sup>4</sup>, subsequently confirmed for the 2009 examinations, found that little mathematics was being assessed in GCSE science papers. This was a concern to the SCORE partners and so they decided to carry out similar research at A level.

The focus of the SCORE research is different from that in this report. It concentrates on the assessment, and so typically examination papers, rather than the syllabus requirements or their relationship to classroom practice. A further difference is that the SCORE research is about all mathematics in science A levels, and not just statistics.

In the SCORE research, teams of subject experts look at a sample of recent examination papers, identifying any mathematics and then classifying it by topic, the number of steps involved (giving a measure of difficulty), complexity, familiarity, relationship to the subject and available marks.

### 1.3.3 The Nuffield Foundation

Complementing the SCORE research into the mathematics in science A levels, the Nuffield Foundation is carrying out a similar exercise for six other subjects: computing, psychology, sociology, geography, economics and business studies. The report on this work has yet to be published.

The Nuffield project follows the SCORE methodology. However, statistics plays a much larger role in some of the subjects involved. The description used in analysing the statistics in the assessment of these subjects is much more detailed.

### 1.3.4 RSSCSE

The Royal Statistical Society Centre for Statistical Education (RSSCSE) runs CensusAtSchool, a major international project in which students provide data about a wide variety of aspects of themselves and their lives. This has produced a very large bank of freely available data, drawn from 10 countries (Australia, Canada, Ireland, Japan, Namibia, New Zealand, South Africa, South Korea, UK and USA) over 11 years, that can be used for teaching and learning. In New Zealand the project has been used as input to the design of the country's new (2008) Mathematics and Statistics curriculum.

The RSSCSE is currently investigating the opportunities for using the CensusAtSchool data in the full range of subjects and across all age groups. Their work has contributed to several aspects of this report, particularly Table 21 (Opportunities for using statistics across the GCSE curriculum).

## 1.4 Scope

This report looks at the statistics taught and learnt in our schools and colleges and so it covers an age range of about 4 to 18. It does not consider university level work.

The main focus is on the effectiveness of the curriculum structures through which statistics is taught. This is linked to the content of a variety of courses and the associated processes within a statistics cycle.

Pedagogy is closely related to the processes fostered by the curriculum and to that extent it is considered here. However, teaching materials and approaches are outside the scope of this report and so have not been considered.

The remit did not extend to the subject of the relationship between digital technology, the driver of the boom in data, and how statistics should be taught. It is, however, inevitable that sooner or later a major piece of research and development work will be needed in this area. This could build on *Digital technologies and mathematics education*<sup>5</sup>, a recent report from the Joint Mathematical Council (JMC).

The saying ‘What you test is what you get’ has never been more true and so assessment is an inevitable part of the background to much of the report. Examples are given of assessment practice in a variety of subjects and how it is expected to interact with what happens in the classroom.

Statistics is an interdisciplinary subject and this means that it does not have a natural home in our curriculum. Particular elements of it are used in many subjects and this is increasingly true the older the age group concerned; at university a substantial majority of undergraduates are on courses in which statistics is embedded. In schools and colleges it is treated as part of mathematics. The report looks in some detail at the issues that this raises.

In this report, several terms are used in their broadest sense.

- ‘Statistics’ includes all relevant techniques.
- The term ‘examination board’ covers awarding bodies.
- The word ‘syllabus’ covers examination specifications.

## 2 Post-16: a developing landscape

The publication of this report, early in 2012, comes at a time of public awareness of the need for greater competence in mathematics in general, and statistics in particular, among the population of the UK. There is a widespread consensus that changes are needed to the provision of these subjects in our schools and colleges, particularly for the post-16 age group. A number of publications and events have been particularly influential.

In July 2010, ACME published a discussion paper, *Post-16 in 2016*<sup>6</sup>, recommending that different pathways should be established to allow all young people to continue with some form of mathematics between the ages of 16 and 18.

Three months later, on 20th October 2010 (World Statistics Day), the Royal Statistical Society launched *getstats*<sup>7</sup>, a campaign for statistical literacy.

In December 2010, the Nuffield Foundation published *Is the UK an outlier?*<sup>8</sup>, a report into the provision of mathematics post-16 in some 24 countries; the data placed England, together with Wales and Northern Ireland, bottom of the table. (Scotland was just a few places higher.) Although not specifically about statistics, this report made the point that in virtually all comparable countries, some form of mathematics is either compulsory to the age of 18 or is embedded in a system that ensures that a large majority of young people take it.

In June 2011, ACME published a two-part report entitled *Mathematical Needs*<sup>3</sup>. One part carried the subtitle *The Mathematical Needs of Learners* and the other part *Mathematics in the Workplace and in Higher Education*. The second of these is particularly relevant. It was based on extensive research, funded by the Nuffield Foundation and the Clothworkers Foundation. This draws on *Is the UK an outlier?* and its first recommendation is

*Policy on mathematics post-16 should ensure that a large majority of young people continue with some form of mathematics post-16.*

However, it goes further than the earlier reports. Based on a large number of interviews with employers and university lecturers it recommends broad content areas for new courses to meet the needs of those who currently give up mathematics at the age of 16. Such students include those going into the full range of higher education courses and into employment. Statistics features strongly among their needs.

In August 2011, the report of Carol Vorderman's task force, *A world-class mathematics education for all our young people*<sup>9</sup> was published. This provides a strategic overview of mathematics education from the start of primary school up to university entrance. It draws on both the Nuffield and ACME reports. It recommends that mathematics should be compulsory to the age of 18, in forms that are appropriate to the students involved. The report emphasises that this will require new courses, in addition to AS and A level Mathematics, with different emphases; statistics will inevitably be an important element in such courses.

## 3 Up to 16: the National Curriculum

From the start of school to the age of 16, much of what young people are taught is determined by the National Curriculum<sup>10</sup>. Teaching it is a statutory requirement for maintained schools. The National Curriculum specifies *Programmes of Study* for each of the Key Stages 1 to 4.

At Key Stage 4 there are two programmes of study for mathematics, one leading to Higher tier GCSE, the other to Foundation tier.

### 3.1 Statistics in the National Curriculum

Under the name *Handling data*, statistics forms one of the four strands in Key Stages 2, 3 and 4 of the mathematics National Curriculum, but is not in Key Stage 1.

The programmes of study for Key Stages 3 and 4 specify the elements in the *data handling cycle*. There are four headings and these are shown in Table 1. Under each of these headings there are several statements, prefixed by ‘Pupils should be taught to:’

Specifying the problem and planning
Collecting data
Processing and representing data
Interpreting and discussing results

**Table 1 Headings for the data handling cycle**

(At Key Stage 2, a single heading of ‘Processing, representing and interpreting data’ is used.)

The data handling cycle is often illustrated in a diagram like that in Figure 2 in Section 4.1, where it is compared to the classification used for statistics in this report.

The National Curriculum also provides a structure for assessing students’ attainment and for monitoring their progress. This is reported as a ‘level’. The requirements are described as *attainment targets*; those for the statistics in each level of mathematics are given in Table 2.

(A quite different use of the term level occurs with the National Qualifications Framework which, for example, describes AS and A level work as ‘level 3’, GCSE as ‘level 2’ and more elementary work as ‘level 1’.)

Level	Content
1	Pupils sort objects and classify them, demonstrating the criterion they have used.
2	Pupils sort objects and classify them using more than one criterion. When they have gathered information, pupils record results in simple lists, tables and block graphs, in order to communicate their findings.
3	Pupils extract and interpret information presented in simple tables and lists. They construct bar charts and pictograms, where the symbol represents a group of units, to communicate information they have gathered, and they interpret information presented to them in these forms.
4	Pupils collect discrete data and record them using a frequency table. They understand and use the mode and range to describe sets of data. They group data, where appropriate, in equal class intervals, represent collected data in frequency diagrams and interpret such diagrams. They construct and interpret simple line graphs.
5	Pupils understand and use the mean of discrete data. They compare two simple distributions, using the range and one of the mode, median or mean. They interpret graphs and diagrams, using pie charts, and draw conclusions. They understand and use the probability scale from 0 to 1. Pupils find and justify probabilities, and approximations to these, by selecting and using methods based on equally likely outcomes and experimental evidence, as appropriate. They understand that different outcomes may result from repeating an experiment.
6	Pupils collect and record continuous data, choosing appropriate equal class intervals over a sensible range to create frequency tables. They construct and interpret frequency diagrams. They construct pie charts. Pupils draw conclusions from scatter diagrams, and have a basic understanding of correlation. When dealing with a combination of two experiments, pupils identify all the outcomes, using diagrammatic, tabular and other forms of communication. In solving problems, they use their knowledge that the total probability of all mutually exclusive outcomes of an experiment is 1.
7	Pupils specify hypotheses and test them by designing and using appropriate methods that take account of variability or bias. They determine the modal class and estimate the mean, median and range of sets of grouped data, selecting the statistic most appropriate to their line of enquiry. They use measures of average and range, with associated frequency polygons, as appropriate, to compare distributions and make inferences. They draw a line of best fit on a scatter diagram, by inspection. Pupils understand relative frequency as an estimate of probability and use this to compare outcomes of experiments.
8	Pupils interpret and construct cumulative frequency tables and diagrams, using the upper boundary of the class interval. They estimate the median and interquartile range and use these to compare distributions and make inferences. They understand how to calculate the probability of a compound event and use this in solving problems.
Exceptional performance	Pupils interpret and construct histograms. They understand how different methods of sampling and different sample sizes may affect the reliability of conclusions drawn. They select and justify a sample and method to investigate a population. They recognise when and how to work with probabilities associated with independent mutually exclusive events.

**Table 2 National Curriculum (mathematics): Handling data attainment targets**

### **3.2 The National Curriculum review**

The research for this report coincides with a review of the National Curriculum. The first stage involves mathematics, science, English and physical education; other subjects will follow in a later stage.

The new National Curriculum for mathematics will not be introduced until 2014, well after the publication of this report, and it will then be some years before things settle down and it is possible to determine its effects. So, inevitably, this report is based on the National Curriculum currently in operation.

Its purpose, however, is to inform an improved statistics provision and so its findings are relevant to the new National Curriculum, and not just in mathematics; this report covers the use of statistics in many other subjects.

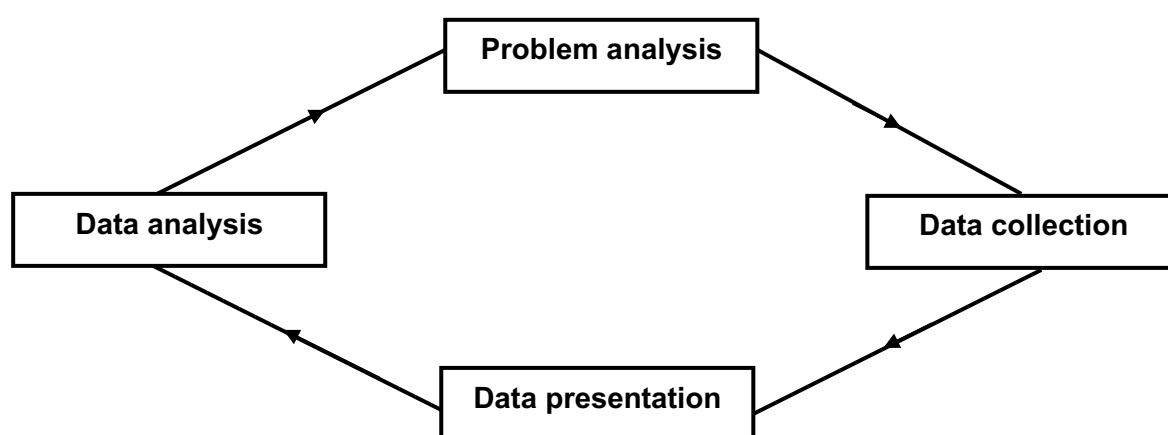
The National Curriculum feeds directly into GCSE syllabuses. While not constrained by it, AS and A level syllabuses naturally build on what is done at GCSE and so are heavily influenced by the National Curriculum. However this should be a two-way process. Those writing the new National Curriculum in any subject should be mindful of the way that it is developing at higher levels and should be aiming to ensure that those students who take the subject beyond GCSE have a suitable background from which to do so. In many cases this is likely to involve greater emphasis on quantitative methods.

## 4 Classification of statistics

### 4.1 The statistics cycle

This report covers the wide variety of procedures and techniques that are used across the school curriculum and can be described as statistics. The question of how best to classify them is not straightforward. Just listing the content in the various syllabuses requiring some statistics would not be helpful to anyone seeking to convert the findings into policy; it would contain too much detail. Instead a classification is needed that describes the present provision in a way that highlights its essential features, including any strengths and weaknesses.

So in this report the statistics is expressed in terms of four broad processes which between them describe the cycle of a complete statistics investigation to solve a problem. This is illustrated in Figure 1; in the rest of this report it is referred to as the *statistics cycle*.



**Figure 1 Classification of statistical work: the statistics cycle**

These processes are described in Table 4 on the next page.

At several places through this report, Figure 1 is reproduced with the various boxes coloured in red, yellow or green. The meaning of this is described below in Table 3.

	Learnt to an appropriate level by most students, given a good teacher
	Coverage for only a few students, or for most but only lightly
	Encountered by very few, if any, students

**Table 3 Colour coding used in this report**



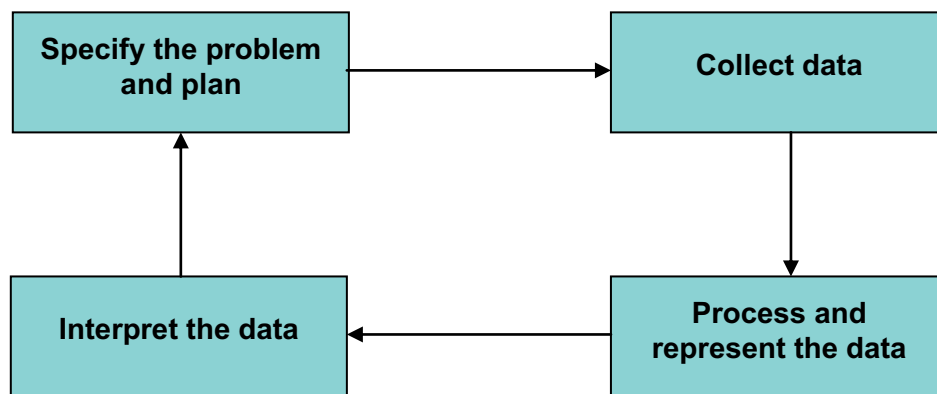
Process	Description	Topics
Problem analysis	<p>This process involves the work at both the start and the end of statistical problem solving. It begins with the analysis of a problem and the design of the proposed approach to tackling it; essential decisions are needed about what data need to be collected and how they will be used. In the subsequent stages the data are collected, presented and analysed.</p> <p>Finally the outcomes are considered in relation to the original problem analysis, and the modelling involved. The conclusion may be that a satisfactory solution to the problem has been found, or it may be a recognition that the approach taken has not been satisfactory; the interpretation may not make sense, or it may not provide sufficient discrimination to be useful. In cases where the problem has not been solved satisfactorily, it will usually be necessary to repeat the whole cycle.</p>	Modelling Errors Risk
Data collection	<p>This process covers the work undertaken when students collect their own (primary) data. Decisions have already been made as to what data need to be collected. It includes deciding how the data will be collected, and then carrying it out. The outcome of this stage is a set of original data.</p> <p>This process also covers the work in sampling from extensive sets of secondary data, such as those on large databases.</p>	Experimental design Sampling techniques
Data presentation	<p>This stage begins with data, which may be primary or secondary, and involves the application of a variety of descriptive techniques and their informal interpretation. Typical examples are grouping and tabulation, display diagrams and simple statistical measures (such as mean and standard deviation, and index numbers).</p>	Tabulation Data display Statistical measures Relative frequency as probability Index numbers
Data analysis	<p>The work in this stage involves mathematical analysis, leading to some inference that is relevant to the problem. Such work often requires an understanding of probability. This stage ends with results that can be applied to the original problem.</p> <p>In simple situations, the data presentation may be sufficient to allow the problem to be solved. In such cases it may be possible to bypass this stage.</p> <p>Much of the statistics in higher education courses comes under this heading.</p>	Use of distributions Statistical inference Hypothesis testing Applying probability

**Table 4 Processes in the statistics cycle**

Figure 1 illustrates the processes but not the depth to which they are being carried out. Obviously there is a progression in sophistication of the work carried out from primary school through GCSE and on to A level. Rather than attempting to show this extra dimension graphically, summaries of typical techniques used by different age groups, and in different courses, are given in the relevant text in Sections 5, 6 and 7.

Several other subjects, for example the sciences and business studies, use similar cycles to represent their activities.

Similar ideas also underlie the data handling cycle in the programmes of study in the National Curriculum for mathematics; however the divisions used in this report in the statistics cycle are not the same as those in the National Curriculum. The data handling cycle is stated in words but is usually interpreted as a summary diagram like that in Figure 2.



**Figure 2 The data handling cycle**

The data handling cycle provides a description for work that is essentially quite simple.

- It effectively goes straight from data presentation to interpretation, thus omitting the process referred to in the statistics cycle as data analysis, where the subject becomes more mathematical.
- It splits the process of problem analysis into initial planning and final interpretation, so does not emphasise the cyclical modelling nature of statistical investigation.

## 4.2 Teaching the full statistics cycle

### 4.2.1 The importance of the statistics cycle

The statistics cycle raises two related questions.

- Does the full cycle really matter?
- How can we ensure that students learn the processes in the full cycle?

The answer to the first question may be summed up by the saying ‘Rubbish in, rubbish out’. Statistics allows us to engage with, analyse and solve a wide variety of problems that are relevant to almost every aspect of our lives. Its importance lies in its usefulness as a practical subject. However, it is only of any value if the data being used are relevant and reliable.

Ensuring the data are relevant requires problem analysis; their reliability depends on the use of appropriate data collection techniques. Without due attention to these processes, there is every possibility of emerging with the wrong answer to a problem, even though the next two stages, of data presentation and data analysis, may have been carried out to perfection. So, the full cycle really does matter. It is fundamental to why statistics is so important.

That leads on to the second question of how best to ensure that students see the need for the various processes and can apply them successfully. Conventional wisdom in this country suggests that if you want students to know something you must assess it. While this may not always be the case, it is probably close to the truth. So some emphasis is placed on assessment throughout this report.

There are a number of assessment techniques that are currently in use in one subject or another that make it highly likely that students in those subjects do work covering the full cycle. Examples of these are given at various points in this report, as they arise.

### 4.2.2 Coursework

Students usually do a piece of *coursework* largely unsupervised and in their own time (see Example 10, page 43). Their write-up is assessed according to criteria laid down by the examination board. The marking is carried out by the teacher. When GCSE was introduced in the late 1980s, coursework was a requirement in most subjects but it fell into disrepute and, following a QCA report in 2006<sup>11</sup>, it was discontinued at GCSE.

There was concern about cheating, with the suspicion that a lot of coursework was done by parents or older siblings; it was said that students were able to buy it from the internet. A different worry was that, instead of being the intended learning experience, coursework was often a mechanistic exercise in which students did the minimum necessary to fulfil the marking criteria; consequently it was not always seen as valuable by teachers. There are, however, various level 3 syllabuses where coursework is still used.

### 4.2.3 Controlled assessment

The term *controlled assessment* is used to describe a variant on coursework that is designed to overcome the problems associated with it. Students choose, or are given, a task which they typically spend some weeks working on. This preparatory work is followed by a specified time under examination conditions when they do their write-ups. 'Controlled assessment' is a technical term and the way it is carried out must comply with regulations<sup>12</sup>.

Controlled assessment is used in several subjects at both GCSE and A level. It is a requirement in GCSE Statistics (see Example 6, page 28) but is not used for GCSE or A level Mathematics.

### 4.2.4 The style of examination questions

Several styles of current examination question are designed to ensure that students actually do some practical work involving data collection during their courses. Examples of all of these are given later in the report. Various levels of pressure are involved. At one extreme is the style of question (see Example 11, page 44) in which candidates are asked to write about a piece of work they have done and are required to submit their work with their examination script.

The next level down is to ask much the same question but not to require candidates to submit their work (see Example 12, page 45). In that case it would be possible to write about a fictitious piece of work but there would be obvious dangers in doing so. So it is very unlikely that any teachers would take the risk of not ensuring that their students had done suitable work.

Finally, there are questions that ask students about designing and conducting statistical experiments but do not refer to work that they have done (see Example 13, page 45, and Example 14, page 46). Although there is no requirement for candidates to have done any practical work, those with background experience of it are at a clear advantage.

All of these questions are likely to involve non-numerical responses and in some cases essays. Although common in other subjects, this style is not within the traditions of mathematics examining; their use in mathematics examinations would require a change of culture.

### 4.2.5 Comprehension tasks

Comprehension tasks are used in the assessment of a number of level 3 qualifications. Candidates are given an article to read, either at the time of the examination or as pre-release material, and are then asked questions on it. Some tasks in the past have concentrated on the problem analysis part of the statistics cycle (see Example 8, page 35).

## 5 Statistics in primary schools

The pattern of education in primary schools differs from that at all subsequent levels in that classes are taught by the same teacher for all subjects. This makes it much easier for cross-curricular work to take place. Teachers know just what their pupils have learnt in any subject area and so it comes naturally to them to relate it to other areas.

Consequently, many children have a richer experience of statistics than might be inferred from looking at the National Curriculum, where it is referred to as data handling and falls within mathematics. Up to the age of 6, children are in Key Stage 1 and the National Curriculum's programme of study for mathematics includes no data handling. It only comes into the programme of study at Key Stage 2 when children are 7 years old.

Reality, however, is quite different. Even in reception classes, children take part in activities which are essentially statistical. Although many of them are not yet able to read and do not know how to write numerals, they learn to record numbers of objects by drawing a line for each one, as they will do later for tally charts. They also record numbers of objects with pictures (or stickers) of each one, i.e. simple pictograms; before long the pictures can be replaced by squares, placed next to each other in lines, i.e. bar charts. Children also learn to sort objects according to particular characteristics (for example by colour or by type of animal) leading on to Venn diagrams, and they learn to group objects.

It is thus the case that a lot of statistical ideas are learnt in Key Stage 1 even though there is no data handling in the programme of study. Paradoxically much of this work is required in the mathematics attainment targets at levels 1 and 2.

At Key Stage 2, when children are aged from 7 up to 11, the programme of study does include some data handling; however, at least some of the work will already be familiar to them. Most, if not all of this, would belong in any worthwhile curriculum and so if it were not placed in mathematics it would have to be found a home somewhere else.

Topic	Notes
Information	Tables, lists and charts
Graphs and diagrams	Pictograms, bar charts and line graphs
Measures	Mode and range for average and spread
Data	Discrete and continuous data
Probability	Likelihood, i.e. probability expressed informally

**Table 5 Techniques in the programmes of study for Key Stages 1 and 2 (to level 4)**

The Key Stage 2 programme of study covers the statistics to about level 4 of the National Curriculum but in fact many pupils (about 35%) go on to level 5 and a few beyond that. At this point some more mathematical requirements are introduced, for example pie charts, the mean and median and the numerical representation of probability.

There are several key features of the work covered under data handling; they include its use in other areas of the curriculum, in cross-curricular projects and in support of the rest of the mathematics curriculum. Many primary teachers find opportunities across the curriculum to show how the data handling techniques that they have taught in mathematics can be used to advantage in other subject areas, such as science and geography. Children are expected to collect, record and display experimental results because doing so is important in these subjects. Thus a major part of the rationale for teaching topics such as bar charts, line graphs and scatter diagrams is to enhance and enable the rest of the curriculum.

### Using Statistics in a Primary School

At The Academy of Trinity we carried out an investigation into punctuality. The focus for the week was decided and we felt that this would be the best time to look at why some children may be late in each of the classes. We decided in groups how we could investigate this issue and how we could improve it. We agreed a set of questions we would ask.

At the start of each day we noted down how many children were late and from which class; we used tally charts to keep a record of these each day. It was interesting standing on our school gate; just doing this made a difference! We then interviewed children to research reasons behind lateness. Using EXCEL, we published our results and created different graphs and tables to interpret the results.

This showed us which class had the highest average of lateness through the week and indeed the number with excellent punctuality. We were also able to see the relationship between the reasons for children being late and the age of these children. There was a considerable amount of work carried out as a group. The tally charts, pie charts, graphs and tables made it easier for us to explain our findings, especially when we wanted to show our results to parents, teachers and children.

From all our gathered data we wrote our research investigation to present to the class; we made it clear what we were investigating and what we found out. We discussed how we could as a school improve the lateness, especially at the start of the week and what has the biggest impact on children being on time. A big success for the school is we can see exactly what incentives helped to improve punctuality. This was a much better exercise than some of the data handling tasks we have been given in the past because it was involving real people and gave us a real focus when collecting our data. Our results would be real and the outcome would have an impact on our school. It was a really worthwhile activity because it mattered to us!

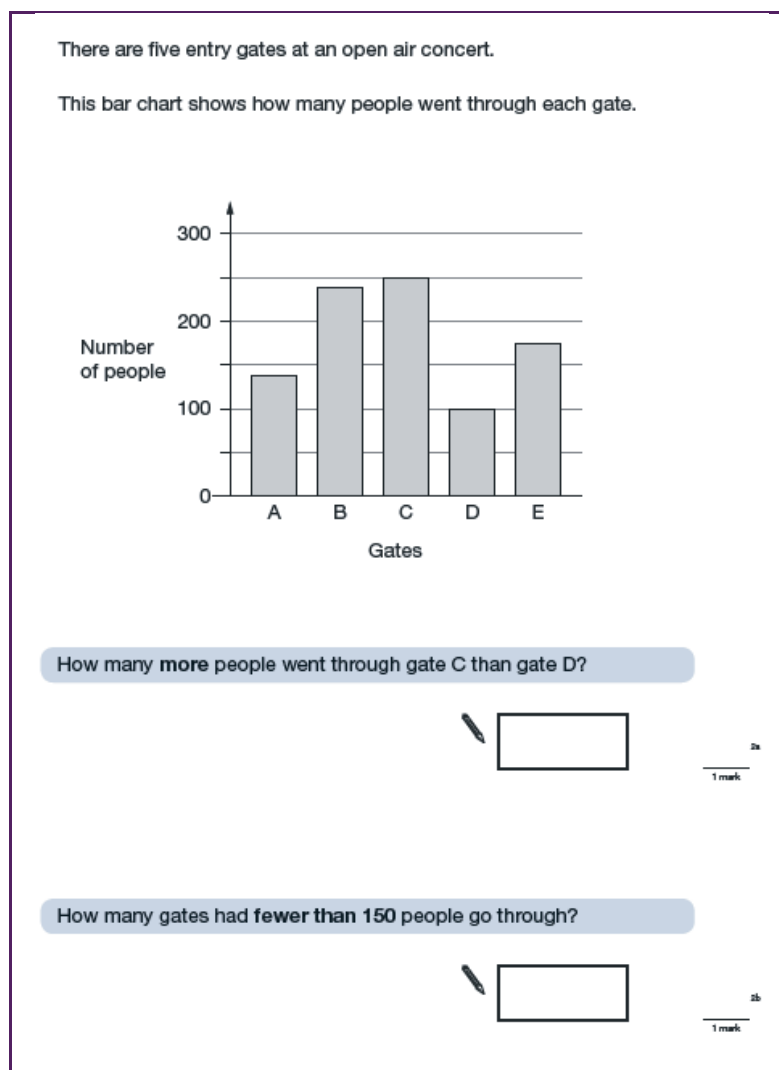
*Thomas Howard, Year six pupil*

#### Example 1 Statistics used in cross-curricular work in a primary school

It is common for a class in a primary school to undertake a major cross-curricular piece of work. Such a project might, for example, be a traffic survey, a study of the local environment or an investigation into pupils arriving late for school. Work from many different subject areas may be involved: English, science, geography, ICT, mathematics and, of course, statistics. At their best, projects like these provide a rich learning experience for pupils.

Much of the work covered under the name of data handling provides an opportunity to practise basic mathematics. Children in Key Stage 1 may be asked to look at their pictogram and work out how many more cats than rabbits are represented, reinforcing their basic arithmetic. In Key Stage 2 those doing some level 5 work meet pie charts and they provide practice with fractions, as does the use of scales in graphs. In the hands of a skilful teacher, work called data handling doubles up as a vehicle for basic numeracy.

A further, and very important, aspect of the statistics covered in primary mathematics is that the key life skill of interpretation is often involved. This is the intention of some of the questions currently set in the National Tests (SATs).



Example 2 A National Test (SAT) question at level 3<sup>13</sup>

A shop sells books, CDs and DVDs.

This pie chart shows the sales of each in one week.

Estimate the **fraction** of the total sales that were DVDs.

2/5  
1 mark

In this week, 200 CDs were sold.

Estimate how many books were sold.

200  
1 mark

**Example 3 A National Test (SAT) question at level 5<sup>14</sup>**

Until recently, advice on the National Curriculum was provided through the National Strategies' *Framework*. Although the framework is not statutory, many teachers are reluctant to depart from it and so regard it as mandatory. The framework puts data handling and measurement together and recommends that they be given six weeks of mathematics time each year<sup>15</sup>. This has created problems.

So much time is neither necessary nor well used. The skills involved in measuring the length of a pencil or drawing a bar chart are elementary and are used in other subjects; they do not need to be revisited for weeks of mathematics every year. There is thus scope for some increase in the content expected at this level, but there should also be an expectation that all schools will undertake valuable cross-curricular work like that in Example 1 and this should be written into the revised National Curriculum.



In many classes, the excessive time allocation written in the framework results in wasteful lessons spent in activities of no mathematical value, for example, colouring in bar charts. This is time taken away from important number work when children are at an age when mastering this is critical for their subsequent development in mathematics and statistics.

The advice given in the framework clearly needs to be rewritten. It would be helpful and realistic if the new National Curriculum could be framed in such a way that, once taught in mathematics, the various topics that make up data handling (and measurement) are seen to belong to the whole primary curriculum, not just to mathematics.

However, the problems caused by the advice in the framework should not be allowed to detract from the richness of experience that data handling makes available to our primary school children.

The statistics that pupils encounter in primary schools is summarised in Figure 3. However, a note of caution is appropriate. There is considerable variability among the many primary schools in the country and not all of them would be well represented by this diagram; there are schools in which only data presentation is given serious attention.

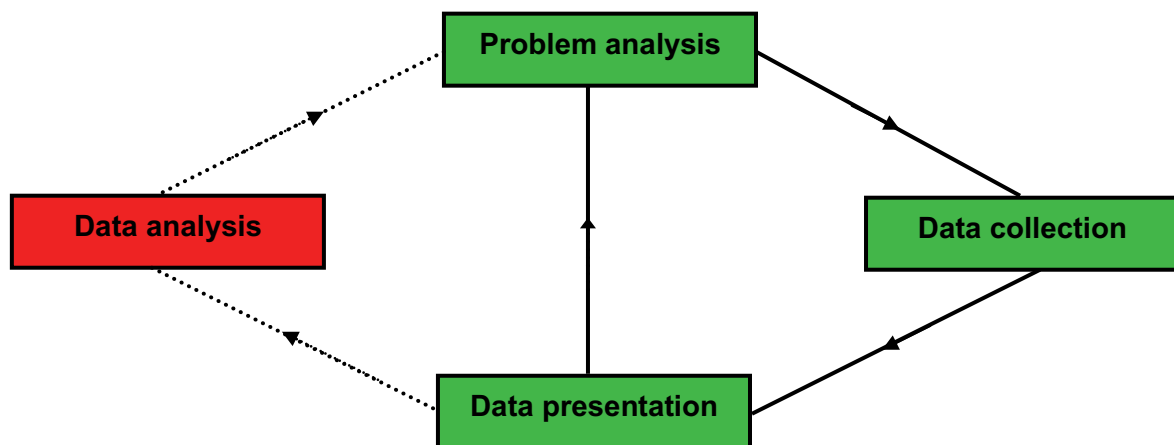


Figure 3 Statistics in primary school mathematics

## 6 Students aged 11 to 16

Most students enter secondary school at the age of 11. In some schools, Key Stage 3 students carry out investigational work, which may involve data collection; however, many do not benefit from such good practice.

Typically, after three years of Key Stage 3, they move on to Key Stage 4 and spend the next two years working for GCSE which they take at the age of 16. Throughout Key Stages 3 and 4, the various subjects are taught by specialist teachers in dedicated lessons. Statistics is taught as part of mathematics.

GCSE students also meet statistics in other subjects and this is described in Section 6.3

Statistics is also a GCSE subject in its own right and this is covered in Section 6.2.

### 6.1 GCSE Mathematics

Almost all students, of the order of 700,000 per year, take courses leading up to GCSE Mathematics. There are two main levels (tiers) of GCSE: Higher and Foundation. (In addition there are 'Entry level' courses which are at a more elementary level; that in mathematics is taken by about 2% of the cohort.)

There are several different ways in which the mathematics and statistics requirements are specified.

- The programmes of study of the National Curriculum state what students should be taught.
- The attainment targets (see Table 2, page 12) state what is required for the different National Curriculum levels.
- The GCSE syllabuses state what may be assessed.
- The GCSE examination papers provide the actual assessment.

Although these ought to be mutually consistent, in practice this is not always the case.

#### 6.1.1 GCSE assessment

The programmes of study for Key Stage 4, both for GCSE Foundation tier and Higher tier, specify the data handling cycle and a number of techniques. The techniques required are summarised in Table 6.

Foundation tier	Extra for Higher tier
<b>Data display</b>	
Frequency diagrams <ul style="list-style-type: none"> <li>• Bar charts</li> <li>• Pie charts</li> </ul>	<ul style="list-style-type: none"> <li>• Histograms</li> <li>• Cumulative frequency graphs</li> </ul>
Stem-and-leaf diagrams	Box-and-whisker plots
Line graphs for time series	
Scatter diagrams <ul style="list-style-type: none"> <li>• The idea of correlation</li> <li>• Line of best fit drawn by eye</li> </ul>	
<b>Summary measures</b>	
Mean, mode and median	Quartiles Moving averages
Range	Interquartile range
<b>Probability</b>	
Probability derived from <ul style="list-style-type: none"> <li>• theoretical models</li> <li>• relative frequency</li> </ul>	
Mutually exclusive events	Tree diagrams

**Table 6 Summary of techniques in the Key Stage 4 programmes of study**

The information about techniques required in the programmes of study is repeated, often in greater detail, in GCSE syllabuses, and is tested on examination papers. It also appears in the attainment targets (see Table 2) for the different National Curriculum levels.

The same level of consistency does not, however, occur for the data handling cycle.

Taken as a whole the data handling cycle is about doing a complete statistics investigation from start to finish and so includes practical work: planning and collecting data. In the past it was covered by a compulsory coursework assignment but this was discontinued in 2007. At that time it was claimed that new styles of examination questions would test the same skills but so far there is no evidence of this happening. Indeed it is hard to see how examination questions in themselves can replicate the hands-on experience of students collecting and working with their own data.

Example 4 shows a typical GCSE question on summary measures.

The number of matches in each of 50 boxes is summarised in the table.

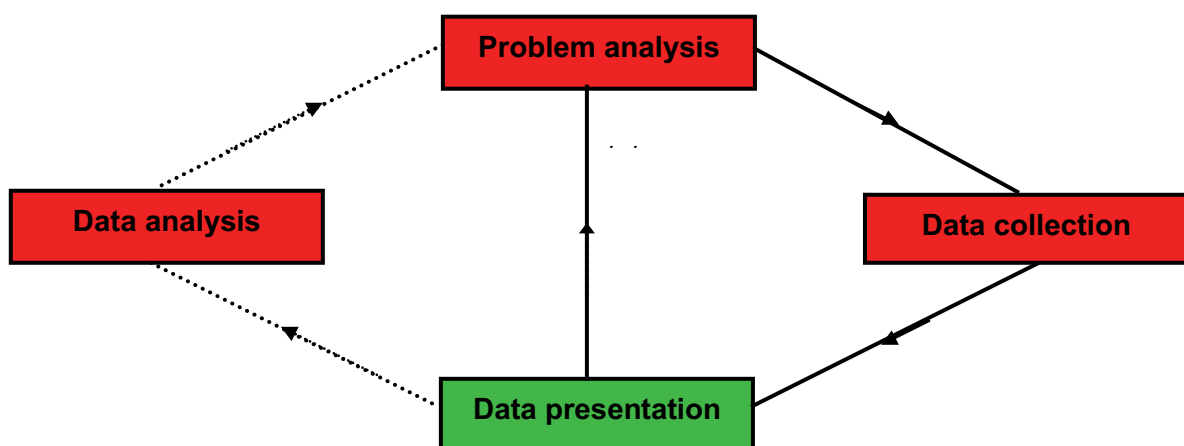
Number of matches	Frequency
46	7
47	18
48	14
49	10
50	1

Calculate the mean number of matches in a box.

**Example 4 A grade C GCSE Mathematics question<sup>16</sup>**

New GCSE syllabuses were introduced for first teaching in September 2010 with the intention that there would be greater emphasis on problem solving. At the same time a pilot began of twin GCSEs; one of these, *Applications of Mathematics*, places particular emphasis on such work. It may well be that these developments result in improved GCSE questions but candidates will still not be working with their own data.

The techniques covered in GCSE Mathematics are all related to presentation rather than analysis; (this contrasts with those in A level Mathematics). Consequently virtually all of the statistics covered in GCSE Mathematics consists of data presentation, as illustrated in Figure 4.



**Figure 4 Statistics in GCSE Mathematics**

## 6.1.2 Coursework in GCSE Mathematics

From 2003 to 2007, a piece of statistics coursework was a requirement of GCSE Mathematics. An extract from one examination board's guide for teachers is given below.

### Coursework Guide – Data Handling Project

During the course candidates should be given the opportunity to develop the knowledge, skills and understanding contained in AO4 (handling data) through project work.

...

The assessment criteria for data handling projects for AO4 are sub-divided into three areas.

These strands are:

Strand 1 – Specify the problem and plan

Strand 2 – Collect, process and represent data

Strand 3 – Interpret and discuss results

...

It is essential that candidates be offered starting points to projects that allow them to plan and make their own decisions. Candidates are expected to define the key issues involved from the starting point and set up a plan of action to solve the problem. Consequently, the starting points listed below are purposefully vague.

1. Each day there are a variety of different newspapers on sale. Why do people read the newspapers they do?
2. Do the candidates at your school have Extra Sensory Perception?
3. Some people think the National Lottery is fixed. Investigate.
4. 'Most cars are red.' Is this true?
- ...
14. Are people's height and mass related?
15. Which second-hand car gives the best value for money?

#### Example 5 Extract from an examination board's former coursework guide<sup>17</sup>

As already described, GCSE coursework fell into disrepute. In addition to the general concerns, a particular problem with the statistics coursework was the practice that evolved of some examination boards providing data to accompany their suggested tasks. As a consequence, those tasks did not fulfil the purpose of ensuring that students collected their own data.

## 6.2 GCSE Statistics

The suite of qualifications designed for 16-year-olds has long included one in statistics. At one time there was O level Statistics and this was duly replaced by GCSE Statistics in the late 1980s. The initial design of the GCSE was based on a strong underlying philosophy, as stated in the Midland Examining Group (MEG) syllabus of that time.

*Statistics is essentially a practical subject ... A basic principle of statistical work is getting information from an appropriately drawn sample and inferring results ... It follows that no course in statistics will convey the essence of the subject if it does not include practical work ... It is hoped that the bulk of teaching of this GCSE course will be based on practical work.*<sup>18</sup>

The current GCSE Statistics syllabuses attempt to retain these ideas. Candidates are required to carry out at least one task under controlled assessment conditions; the task is provided by the examination board but is sufficiently general to allow students to make individual responses.

### ‘Where do the wealthiest people live?’

Using the above question and your knowledge of statistical methodology, design an appropriate hypothesis to test. You should then investigate the hypothesis and draw a conclusion.

Your investigation should be restricted to the United Kingdom.

#### Example 6 A specimen task for controlled assessment in GCSE Statistics<sup>19</sup>

The syllabuses of those examination boards that offer GCSE Statistics are very similar in content. In Table 7 this is contrasted with that in GCSE Mathematics.

Description	GCSE Statistics syllabus requirements
Overlap with GCSE Mathematics	All the statistics in GCSE Mathematics is included but not always in the same tier.
Filling out GCSE Mathematics	The Statistics syllabuses contain topics that could be considered implicit in the data handling cycle in GCSE Mathematics but are not well expressed there (and so are unlikely to be examined), particularly in relation to planning an investigation, types of data, sampling methods and sampling constraints.
GCSE level material that is not in Mathematics	The Statistics syllabuses contain a number of topics that are widely used; some of them could well be placed in GCSE Mathematics. Examples include time series, weighted means and index numbers.
Extension material	There are a number of topics that are definitely beyond those in GCSE Mathematics. Examples include standard deviation, Spearman’s rank correlation coefficient and the Normal distribution.

**Table 7 Comparison of statistics content in GCSE Mathematics and GCSE Statistics**

In summary, GCSE Statistics provides a framework that can cover the full statistics cycle, although only lightly in terms of data analysis. This is shown in Figure 5.

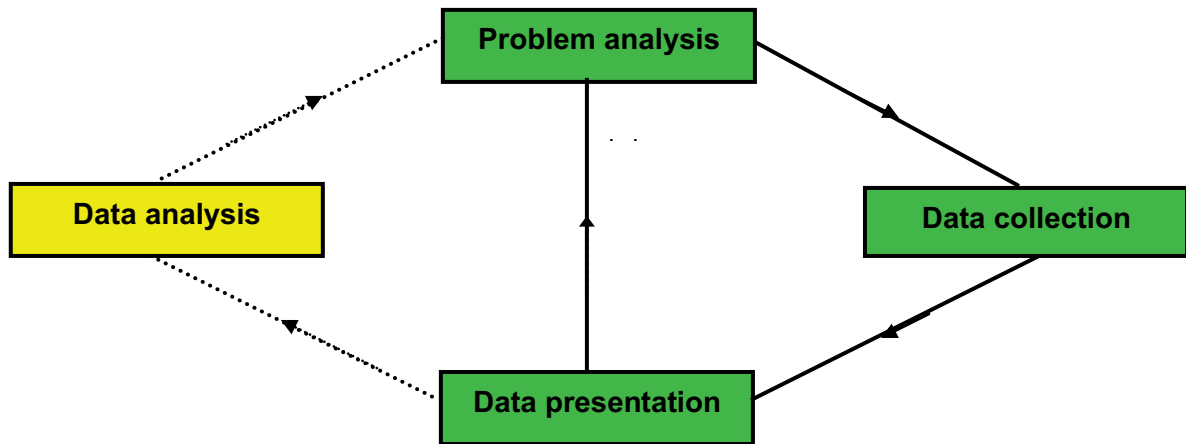


Figure 5 Statistics in GCSE Statistics

However, there is anecdotal evidence from teachers that the potential of this syllabus is often not matched by reality. This is supported by Figure 6, which shows the uptake of GCSE Statistics in recent years; the very large variation suggests that factors other than an appreciation of statistics are involved.

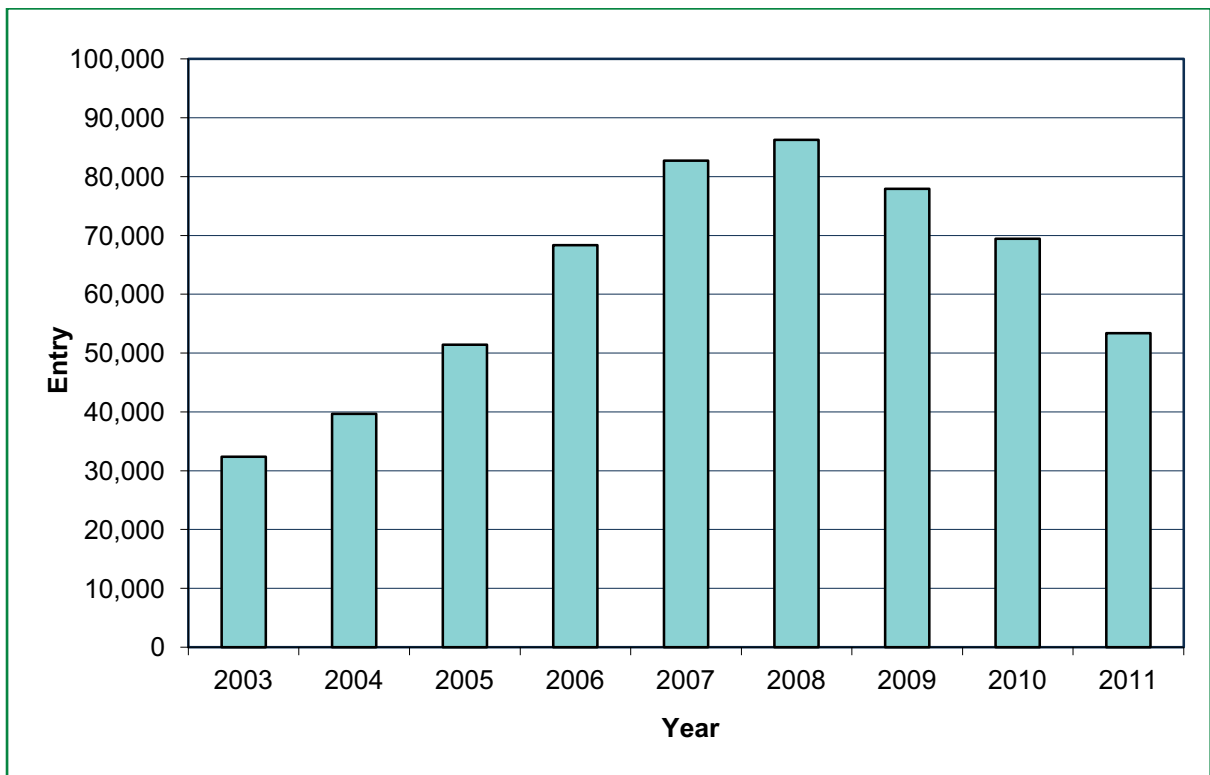


Figure 6 Entries for GCSE Statistics<sup>20</sup>

Given the considerable overlap in content with Mathematics, it is suggested that many schools see it as a relatively easy way for students to obtain an extra GCSE and so to make a greater contribution to performance table rankings. A contributory factor in this may be the increasing trend towards earlier entry in mathematics, leaving some students in need of a different course to take in Year 11.

## 6.3 Statistics in other subjects at GCSE

The opportunities for using statistics in subjects other than mathematics in the years up to and including GCSE were highlighted in 2000 in *Statistics Across the English National Curriculum*<sup>21</sup> and more recently in the Royal Statistical Society's response<sup>22</sup> to the call for evidence for the National Curriculum Review in 2011.

The situation is not static. The increasing emphasis on the quantitative aspects of most subjects should be feeding down from higher education and employment into A level and GCSE syllabuses and their teaching. To assist this process, Table 21, at the end of this report, gives the assessment requirements and also teaching and learning opportunities for the major GCSE subjects.

Meanwhile, in the present provision at GCSE level, the science subjects and geography stand out as providing particularly good opportunities for students to use statistics.

### 6.3.1 Science subjects

At GCSE, Physics, Chemistry and Biology may be taken as separate subjects or they may be combined as Science and as Additional Science. The subject criteria for all of these subjects include a list of required mathematics. The statistics topics are given in Table 8.

Foundation tier	Extra for Higher tier
Calculate arithmetic means	Understand and use percentiles and deciles
Plot and draw graphs (line graphs, bar charts, pie charts, scatter graphs, histograms) selecting appropriate scales for the axes	
Extract and interpret information from charts, graphs and tables	
Understand the idea of probability	

**Table 8 Statistics in the mathematics requirements in GCSE sciences<sup>23</sup>**



The subject criteria also state the following requirement. (The wording is the same for all these subjects except Science where it is a little less demanding.)

*Develop hypotheses and plan practical ways to test them including risk assessment; manage risks when carrying out practical work; collect, process, analyse and interpret primary and secondary data including the use of appropriate technology to draw evidence-based conclusions; review methodology to assess fitness for purpose, and review hypotheses in the light of outcomes.*<sup>23</sup>

In all the science subjects this is fulfilled through fieldwork or practical work and the associated controlled assessment accounts for 25% of the total mark. This work is clearly closely related to that in the statistics cycle, particularly in the case of biology where sampling is likely to be involved.

### 6.3.2 Geography

The subject criteria for Geography require students to carry out fieldwork and out of class learning. This is then the subject of controlled assessment which accounts for 25% of the total mark. Such work produces data which students then typically display and interpret. In doing this many geography students use statistical techniques they have learnt in mathematics, and some go further, for example using Spearman's rank correlation coefficient.

In addition to the controlled assessment, use is often made in geography lessons of data presentation techniques covered in GCSE Mathematics, for example, measures of central tendency (mean and median) and spread (range), pie charts, bar charts and scatter diagrams.

## 7 Post-16 provision

This section looks at what statistics students learn in the post-16 curriculum, and where they do so. Students in this age group have taken GCSE Mathematics and so the various techniques that it covers define the background knowledge they should already have.

Most of the statistics learning by students in this age group occurs within A level courses, either in mathematics or in other subjects. While, in most subjects, there is some variability in the statistics required by the syllabuses of different examination boards, there are also often considerable differences between the syllabuses offered by the same examination board in the same subject. Some examination boards deliberately offer two syllabuses: one that is quite routine and an alternative that is designed to provide students with a rich learning experience.

These alternative syllabuses have often benefited from considerable outside expertise, typically from curriculum development bodies, charitable foundations and subject associations. They tend to place more emphasis on individual investigative work, and in doing so to create more opportunities for the use of the statistics cycle.

### 7.1 A level Mathematics

All the syllabuses for AS and A level Mathematics, and for Further Mathematics, are modular; three modules are needed for AS level and six for A level. Most of the syllabuses have four statistics modules, which are cumulative in content; thus Statistics 4 assumes knowledge of Statistics 3 which assumes Statistics 2 which in turn assumes Statistics 1.

At AS level, two of the three modules must be pure mathematics leaving room for only one applied module, which may be (and often is) Statistics 1. (The other options for the applied module are Mechanics 1 and Decision Mathematics 1.)

Similarly, for A level four of the six modules must be pure mathematics and only two are applied; these may be Statistics 1 and 2, or Statistics 1 and a module from another strand, for example Mechanics 1, or they may both be from other strands. Statistics 3 and 4 inevitably belong to Further Mathematics.

For a number of reasons, available data do not allow an exact determination of the number of people in any cohort who take the various statistics modules; however, the number of students taking AS and A levels in Mathematics is subject to quite large changes (currently increases) from year to year so there would be limited value in such a figure anyway. The best that can reasonably be done is quite a rough estimate.

This is given in Table 9 for the cohort who took their A levels in 2011.

Qualification	Estimated numbers
AS level Mathematics	110,000
A level Mathematics	83,000
Statistics 1	100,000
Statistics 1 & 2	30,000
Statistics 1, 2 & 3	3000
Statistics 1, 2, 3 & 4	250

**Table 9 Estimated numbers taking Statistics modules within AS & A level Mathematics and Further Mathematics in 2011**

The content of the statistics modules varies between syllabuses. Table 10 lists typical content for Statistics 1 and 2. Items in brackets are in some but not all of the syllabuses.

Topic	Notes
Data display	Revision of GCSE Higher tier
Data measures	Revision of GCSE, Standard deviation
Random variables	Discrete, (Continuous)
Distributions	Uniform, Binomial, Poisson, Normal (Approximations)
Bivariate data	Correlation, Regression
Central Limit Theorem	(Confidence intervals)
Hypothesis testing	Binomial, Normal, (Poisson), ( $t$ -test), (Correlation), ( $\chi^2$ )
Probability	Revision of GCSE Higher tier and extension to conditional probability

**Table 10 Typical content of Statistics 1 and 2 modules**

During the 1990s some A level Mathematics syllabuses included statistics coursework, either as an option or as a mandatory part of the assessment. So at that time, there were some mathematics students who encountered this aspect of statistics; for many of them, this was a rich and valuable experience. In 2000 revised syllabuses were introduced under Curriculum 2000 and some of these retained statistics coursework.

Paradoxically, this was undermined by the introduction of statistics coursework into GCSE Mathematics in 2003. For reasons that have already been described, it became common for GCSE students to have a poor experience with their statistics coursework and so to be reluctant to take on similar work the following year as part of AS level Mathematics. Statistics coursework acquired such a bad name that it was withdrawn from AS level or became optional.

Virtually all of the statistics in AS and A level Mathematics syllabuses is now assessed entirely by timed written examinations, and so there is no requirement to design or carry out an experiment. (A partial exception occurs with Statistics 1 in the AQA syllabus which allows candidates to replace an examination question by coursework. Few candidates, less than 2%, take this option.)

A few teachers may, as a matter of good practice, require their students to do such work but they are exceptional. By far the majority of students taking AS or A level Mathematics derive no practical experience of designing or carrying out an experiment from it, or of deciding how to process their own data. They learn to answer examination questions which require them to use particular techniques to analyse small data sets. A typical question is shown in Example 7.

The random variable  $X$  represents the reaction times, in milliseconds, of men in a driving simulator.  $X$  is Normally distributed with mean 355 and standard deviation 52.

- (i) Find
- (A)  $P(X < 325)$ ,
- (B)  $P(300 < X < 400)$ . [6]
- (ii) Find the value of  $k$  for which  $P(X < k) = 0.2$ . [3]

It is thought that women may have a different mean reaction time from men. In order to test this, a random sample of 25 women is selected. The mean reaction time of these women in the driving simulator is 344 milliseconds. You may assume that women's reaction times are also Normally distributed with standard deviation 52 milliseconds. A hypothesis test is carried out to investigate whether women have a different mean reaction time from men.

- (iii) Carry out the test at the 5% significance level. [8]

**Example 7 A typical statistics question in A level Mathematics<sup>24</sup>**

There is nothing intrinsically wrong with questions like this. They assess students' knowledge of the techniques in the syllabus efficiently, and suggest contexts in which they could be used. Such questions will always be needed but they do not ensure that students meet the full statistics cycle. This does not happen at present, and so the experience of students taking AS and A level Mathematics is summarised by Figure 7.

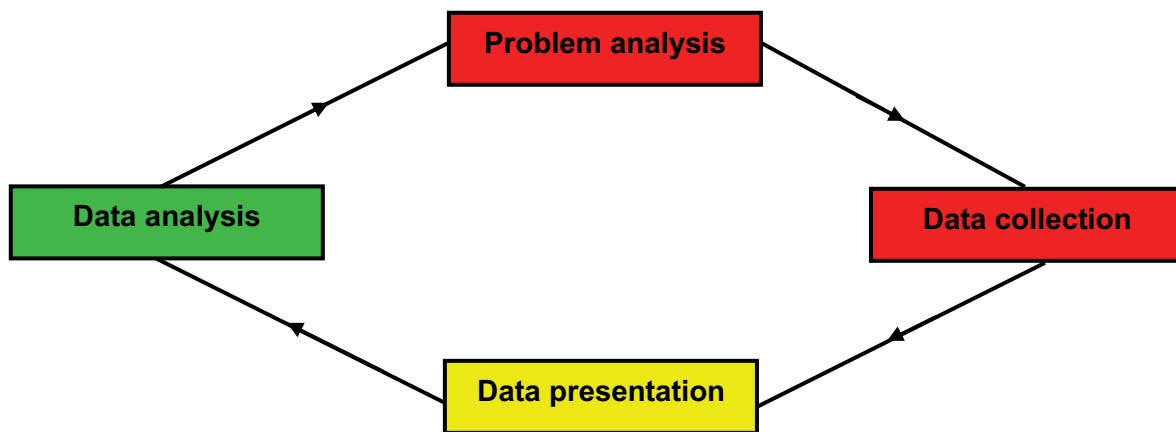


Figure 7 Statistics in AS and A level Mathematics

In one mathematics syllabus<sup>25</sup>, all A level candidates are required to take a comprehension paper. This often requires candidates to read and understand an example of quite detailed problem analysis. Many of those set have been statistical in nature; titles of some of them are given in Example 8.

Comprehension topics	
Card shuffling Screening for rare diseases Trains' punctuality Directional data	Herd immunity Modelling athletics records Credit card fraud Estimating animal populations

Example 8 A sample of comprehension topics used in recent years<sup>26</sup>

While most students taking A level Mathematics do not progress beyond Statistics 1, there are those who take Statistics 2 as well and some Further Mathematics students go on to Statistics 3 and 4. The emphases of the various examination boards' syllabuses reflect somewhat different underlying philosophies. Nonetheless, many topics are to be found in all of them. The statistics available overall is summarised in Table 11.

In 2004, the requirements of A level Mathematics were reduced from three applied modules to the present two. The effect has been to reduce the amount of statistics many students learn, and also what is available; until then, many syllabuses had six statistics modules but this has now been reduced to four. (This change was designed to rectify the problems caused by Curriculum 2000 in which mathematics was seen to be too demanding, with a loss of nearly 20% of A level candidates.)

<b>Data</b>	<b>Hypothesis tests</b>
Measures of location	Language of hypothesis tests
Measures of spread	Type 1 & 2 errors
Data presentation	Binomial test for probability
<b>Samples</b>	Normal test for mean
Sampling	<i>t</i> -test for mean
Samples and populations	$\chi^2$ test (contingency table)
<b>Probability</b>	$\chi^2$ test (goodness of fit)
Laws of probability	Test for Poisson parameter
Conditional probability	Test for correlation
Permutations and combinations	Paired sample test(s)
<b>Random variables</b>	Two-sample test(s)
Discrete random variables	Wilcoxon test(s)
Continuous random variables	Tests for variance
Linear combinations of random variables	Analysis of variance
<b>Bivariate data</b>	Power of a test
Correlation	<b>Estimation and confidence intervals</b>
Linear regression	Estimation
Rank correlation	Confidence interval for mean
<b>Distributions</b>	Confidence interval for variance
Uniform (rectangular) distribution	<b>Other techniques</b>
Binomial distribution	Estimators
Poisson distribution	Generating functions
Normal distribution	
Exponential distribution	
Geometric distribution	<b>Key</b>
Discrete bivariate distributions	In five or six of the six syllabuses
Central Limit Theorem	In three or four of the six syllabuses
Distributional approximations	In one or two of the six syllabuses

**Table 11 Statistics topics in the A level Mathematics and Further Mathematics syllabuses**

## 7.2 AS and A level Statistics

Until 2004 it was possible to obtain AS and A level Statistics qualifications from modules in the mathematics suite. Thus a student could take Statistics 1, 2 and 3 and obtain AS Statistics, and some did so. In those days some syllabuses had six statistics modules and these could be aggregated to give a full A level in Statistics.

Mathematics syllabuses were changed in 2004 and this alternative use of statistics modules was disallowed. Instead Statistics syllabuses were required to have different modules from Mathematics, with a maximum overlap of one module permitted.

At that time, one examination board (AQA) started offering a syllabus for AS and A level Statistics, and two years later Mathematics in Education and Industry (MEI) developed an AS syllabus which is run by OCR. These syllabuses have not had a major uptake; the AQA syllabus attracts about 1000 candidates per year at A level and 1500 at AS level, MEI quite a lot fewer.

However, those responsible for these syllabuses are convinced that there is a national need for them and are frustrated that this does not translate into much larger candidate numbers.

The content of these syllabuses covers similar statistics topics to those in AS and A level Mathematics and in addition some areas of a more practical nature. Examples of such additional topics are experimental design (both) and statistical process control (AQA).

Both syllabuses are designed to be accessible to those who are not taking AS or A level Mathematics. This limits the theoretical work underpinning a number of distributions, the more so since no calculus can be involved. Consequently these syllabuses have a somewhat less theoretical feel to them.

The current regulations allow one module to be common to Mathematics and Statistics, and in both cases this is Statistics 1. The advantage of this is that it allows students to switch between Mathematics and Statistics without having to make a completely new start. The disadvantage is that it might be preferable for the Statistics qualification to start with a less theoretical module than that in the Mathematics strand.

The AQA syllabus includes a coursework option for 25% of the assessment of its first module but very few candidates actually do it. Apart from that, neither syllabus requires students to collect their own data; so they do not cover all the processes in the complete statistics cycle.

### 7.3 Free-Standing Mathematics Qualifications and Use of Mathematics

In the 1990s General National Vocational Qualifications (GNVQs) were developed to meet the changing needs of vocational students. However, their design was criticised for not including enough mathematics, and for the lack of external assessment of what was there<sup>27</sup>. To overcome this deficit a number of mathematics units were developed, and they are now called *Free-Standing Mathematics Qualifications* (FSMQs)<sup>28</sup>.

At Level 3 there is one statistics FSMQ, entitled *Using and Applying Statistics*. It follows on from GCSE and is allocated the same amount of teaching time as an AS module, so in those respects is comparable with the Statistics 1 module in AS Mathematics. Its design, however, is very different.

The assessment is 50% examination and 50% coursework. The examination questions are based on a data sheet which candidates are given about a month in advance; an example is shown in Example 9. This allows more realistic data and contexts to be used, with candidates having time to assimilate them.

The coursework has two equally weighted elements.

- Students carry out an investigation into a situation of their own choosing, using data they have collected. The work is expected both to reinforce their classroom learning and also to extend it beyond the examination syllabus. In their write-up they are expected to use the statistical capability of a spreadsheet.
- In the other piece of coursework students carry out a critical analysis of the statistical work of others. They are told to consider ‘*whether you as reader of the work should accept or reject any reported findings based on the information available to you*’<sup>29</sup>.

As illustrated in Figure 8, this module covers all the processes in the statistics cycle. It is very highly regarded by those teachers who offer it, but its uptake as a stand-alone unit is low, fewer than 500 students per year. That, however, is not the whole story as it can also be combined with other units to form AS *Use of Mathematics*. This adds considerably to the uptake, by about another 1000 students.

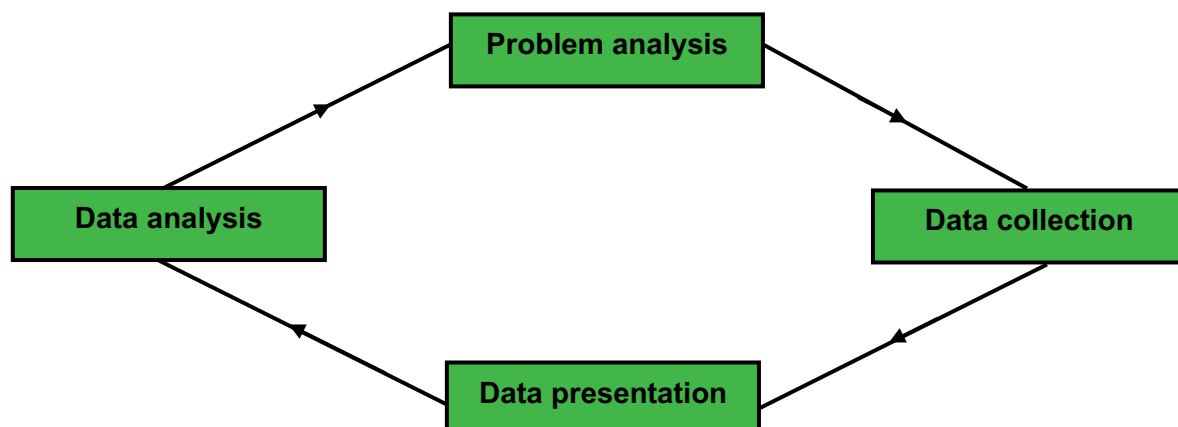


Figure 8 Statistics in the FSMQ Using and Applying Statistics



**Pre-released data**

**Birds**

The table below gives information about the twelve most common garden birds in the UK.

	Number of eggs	Incubation (days)	Fledging (days)	Maximum lifespan (years)	Length (cm)	Wingspan (cm)	Weight (g)
Blackbird	4	13	14	20	25	36	103
Blue Tit	10	14	19	21	11	19	11
Chaffinch	5	13	14	14	14	27	24
Collared Dove	2	16	17	16	32	51	205
Duncock	5	13	12	9	14	20	21
Great Tit	9	14	19	15	14	24	18
Greenfinch	5	13	16	12	15	26	89
House Sparrow	4	13	15	12	15	23	31
Magpie	6	22	27	21	45	56	225
Robin	5	14	14	8	14	21	18
Starling	6	13	21	22	21	40	75
Woodpigeon	2	17	34	14	41	78	449

**Examination question**

- (a) Use your calculator to find:
- (i) the mean length,  $\bar{l}$ , and the mean wingspan,  $\bar{w}$ ;
  - (ii) the correlation coefficient between  $w$  and  $l$ ;
  - (iii) the equation of the line of best fit. *(5 marks)*
- (b) Plot the line of best fit on the scatter diagram on the answer sheet. *(4 marks)*
- (c) Briefly interpret the gradient of the line of best fit in terms of the length and wingspan of garden birds. *(2 marks)*
- (d) The correlation coefficient between wingspan and length for seabirds is 0.811. Explain what this tells you about the relationship between wingspan and length of seabirds when compared with that of garden birds. *(2 marks)*

**Example 9 Part of the pre-release material for one examination of the FSMQ Using and Applying Statistics and the associated examination question<sup>30</sup>**

## 7.4 Statistics in other subjects at A level

Many subjects other than mathematics contain some statistics at AS and A level.

Students typically spend two years in sixth form. It is common to take four subjects at AS level during the first year and then to drop one of them for the second year, but continue with the remaining three for the full A level.

Consequently quite a lot of statistics is learnt outside mathematics. Approximate numbers taking relevant subjects at A level are given in Table 12. The pattern of entry means that numbers for AS level are somewhat larger, with some students not continuing that subject to the full A level.

Mathematics	83,000	Geography	31,000
Further Mathematics	12,000	Environmental Studies	2,000
Physics	33,000	Economics	24,000
Chemistry	48,000	Business Studies	30,000
Biology	62,000	Sociology	31,000
Psychology	56,000	General Studies	41,000
Geology	2,000		

Table 12 A level numbers in 2011 (to the nearest 1000)<sup>31</sup>

### 7.4.1 Science subjects

Biology, chemistry, physics, psychology and geology are all categorised as science subjects and as such their syllabuses are required to state their mathematical requirements, drawing from a standard list. The section of the list entitled *Handling data* is given in Table 13, and its applicability to the different science subjects is in Table 14. The criteria for science subjects state that students ‘need to have been taught, and to have acquired competence in’ these topics<sup>32</sup>.

Statistics in science AS and A levels	
(a)	Use an appropriate number of significant figures.
(b)	Find arithmetic means.
(c)	Construct and interpret frequency tables and diagrams, bar charts and histograms.
(d)	Understand simple probability.
(e)	Understand the principles of sampling as applied to scientific data.
(f)	Understand the terms mean, median and mode.
(g)	Use a scatter diagram to identify a correlation between two variables.
(h)	Use a simple statistical test.
(i)	Make order of magnitude calculations.

Table 13 Extract from the Criteria for AS/A level Sciences<sup>32</sup>

Subject	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
Biology	✓	✓	✓	✓	✓	✓	✓	✓	
Chemistry	✓	✓							✓
Physics	✓	✓							
Psychology	✓	✓	✓	✓	✓	✓	✓	✓	✓
Geology	✓	✓	✓	✓	✓	✓	✓	✓	✓

Table 14 Statistics requirements in particular Science AS/A levels<sup>32</sup>

Science syllabuses are also required to include (and comply with) the statement entitled *How science works*<sup>32</sup>. This consists of twelve statements about science, one of which is ‘Analyse and interpret data to provide evidence, recognising correlations and causal relationships’. So, correlation appears in all science syllabuses.

### 7.4.2 Physics and chemistry

There is extensive practical work in both physics and chemistry which is the subject of controlled assessment at both AS and A level. Students are required to make measurements and draw conclusions from the resulting data. However, the variability in the measurements is usually due to experimental error rather than any inherent variability in what is being measured, as would often be the case in, for example, biology or psychology. (An exception occurs in certain physics experiments that are designed to simulate radioactivity.)

Table 15 gives an outline of the processes followed in physics and chemistry. They may best be described as variants of those in the statistics cycle. A difference is that no sampling is likely to be involved in the data collection undertaken by physics and chemistry A level students.

Both of these subjects have their own well-established methodologies. They are not strongly statistical but nonetheless it could be expected that a student with a good background in statistics would find it helpful; equally students who fully appreciate the philosophy underpinning the experimental work in physics and chemistry are likely to be able to relate it to statistics.

Process	Physics	Chemistry
<b>Problem analysis</b>	At A level, problem analysis is not expected of physics students. They are, however, expected to review their methods in the light of their results. There is a strong emphasis on error analysis.	At A level, chemistry students tend to use well-defined experimental methods. They are, however, expected to review them in the light of the accuracy of their results.
<b>Data collection</b>	Physics students are expected to give some thought to experimental design and the subsequent data collection. However, sampling is rarely required.	The data collected are those produced by the experiments and so no sampling decisions have to be made.
<b>Data presentation</b>	Students use presentation techniques from the statistics in GCSE Mathematics. In addition they use error bars.	Students use some of the presentation techniques in the statistics in GCSE Mathematics to present their data. In addition they use error bars.
<b>Data analysis</b>	Students are expected to use their data to establish algebraic relationships between the variables involved.	Any calculations applied to the data are specific to chemistry and would not be recognised as lying within statistics.

**Table 15 The statistics used in A level Physics and Chemistry**

### 7.4.3 Biology, geography and psychology

Most A level students in biology, geography and psychology design and conduct their own experiments. In all these subjects the assessment arrangements are designed to ensure that students actually do such work.

In biology there is coursework at both AS and A level. Some statistics-rich exemplar tasks are given in Example 10.

A level Biology
Investigating the effect of
posture on vital capacity
mineral deficiencies on the rate and overall height of oat seedlings
aspect on moss distribution on tree trunks
light intensity on stomatal density on laurel leaves

**Example 10 Coursework tasks for A level Biology<sup>33</sup>**

In geography and psychology, students are expected to draw on their experience of field work or practical work in answering examination questions.

The following examples show different styles of questions that are currently being used. They require an understanding of hands-on work covering all or part of the statistics cycle. Consequently most teachers regard such work as an essential part of their students' preparation.

All of these examples are taken from recent examination papers. So they demonstrate assessment techniques which are acceptable to the examination boards.

In Example 11, candidates are required to submit their own geography fieldwork and to answer generic questions about it.

In Example 12, candidates are asked questions about a piece of geography fieldwork they have carried out, but they are not required to submit their actual work.

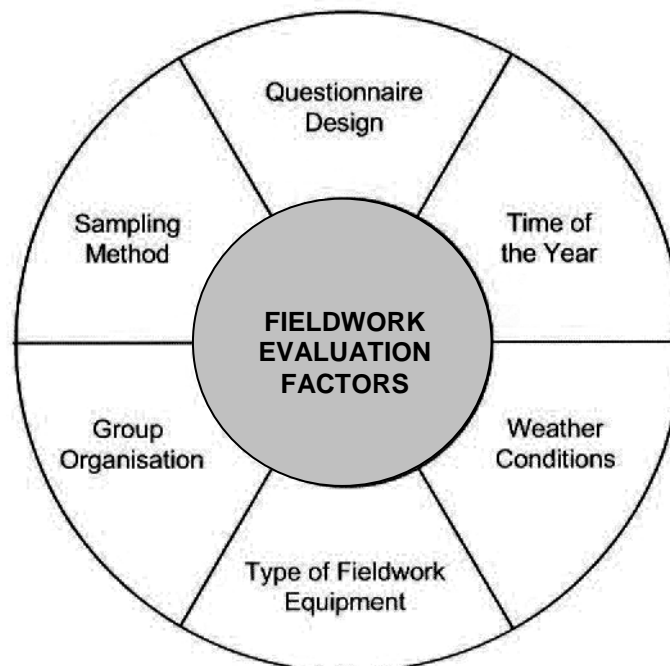
Examples 13 and 14, are both taken from psychology papers. While they do not refer to their work explicitly, it would nonetheless be hard for candidates to answer them confidently if they had not carried out such work for themselves.

Submitted summary of fieldwork and table of data.

**At the end of the examination these should be attached securely to this paper using the treasury tag supplied.**

- 1 (a) "An awareness of hazards/risks and contingency plans is essential when planning fieldwork."  
Discuss this statement with reference to **your fieldwork** planning. [6]
- (b) Select **one** of the following statistical methods which could be used for the analysis of your fieldwork data. Your selected technique must be relevant to your fieldwork aim/hypothesis.
- Spearman's Rank Correlation
  - Nearest Neighbour Analysis
  - Mean, median, mode **and** range
- (i) In the box below apply your selected statistical technique to your data and, if relevant, comment on the statistical significance of the outcome. (Formulae and significance graphs are provided – **Resource 1A** and **Resource 1B** overleaf.) [7]
- (ii) Having completed your statistical analysis, discuss the geographical/theoretical factors which may explain your result in question 1(b)(i). [6]
- (c) Study **Resource 1C**, which shows some of the factors which can be considered when evaluating a field study.

**Resource 1C**



*Source: Principal Examiner*

- Explain how any **two** of these factors may have influenced the nature and reliability of your geographical conclusion. [8]
- (d) Explain **one** way in which your chosen fieldwork location proved to be either suitable or unsuitable for the investigation of the **geographical** aim of your study. [3]

**Example 11 An AS level Geography question**<sup>34</sup>

### Geographical Investigation

Answer **both** of the following questions. You should base your answer on geographical investigation(s) undertaken during your A level course. You may use the same investigation title or a different investigation title for each question.

- 4 Evaluate the relative contribution of primary and secondary data to your geographical investigation.  
State the title of the investigation.
- 5 Evaluate the success of your investigation and suggest how it could be improved.

#### Example 12 Two A level Geography questions<sup>35</sup>

You have been asked to conduct an experiment to test the effectiveness of two different types of revision. One type is doing some each day over an extended period (spaced revision). The other type is where the learner does all the revision in a short period of time (cramming).

Write a plan for an experiment to test which of these two ways of revising is better. You should use either a laboratory experiment, a field experiment or a natural experiment.

You should consider the following issues (there are others):

- design
- variables
- ethical issues
- type of data and how it would be gathered.

(12)

#### Example 13 An A level Psychology question<sup>36</sup>

Read the following material and answer **ALL** the questions that follow.  
You are advised to spend at least 5 minutes reading the information provided.

A laboratory experiment was carried out to see if noise has an effect on memory. An opportunity sample of twenty participants were placed in a quiet room and given a list of ten words to remember in 1 minute. They were then tested to see how many words they remembered. The same group of participants were then placed in a room with loud music playing and given a list of different words to remember. Again participants were given 1 minute and tested afterwards. Results are shown below.

The statistical level accepted was 0.05 and the Wilcoxon test was applied
Observed (Calculated value) 43
Critical (Table value) 52

- (a) Define the term ‘reliability’. [2]
- (b) (i) Identify the experimental design that was used in this research. [2]  
(ii) Give one advantage of the chosen experimental design. [2]  
(iii) Give one disadvantage of the chosen experimental design. [2]
- (c) Explain why the Wilcoxon test was chosen. [2]
- (d) Explain the way in which one confounding variable may have affected this study. [2]
- (e) (i) Describe what is meant by the term ‘opportunity sampling’. [2]  
(ii) Describe **one** advantage of this sampling method. [2]  
(iii) Describe **one** disadvantage of this sampling method. [2]  
(iv) Identify **and** describe another method by which the sample may have been chosen in this research. [3]
- (f) Look at the results above.  
(i) Explain why the experimental hypothesis was accepted or rejected. [2]  
(ii) Explain what is meant by ‘The statistical level accepted was 0.05’. [2]

**Total Marks 25**

**Example 14 An A level Psychology question<sup>37</sup>**



Most A level students of biology, geography and psychology learn and carry out some hypothesis tests. The tests required are given in Table 16 but it should be noted that this gives all of the tests that may be used in any subject and so more than are likely to be encountered by any particular student. Three tests that are commonly used in these subjects are Spearman's rank correlation test, the  $\chi^2$  test and the Mann–Whitney *U*-test, as shown in Table 16. Of the minority of students who do include two statistics modules within A level Mathematics, some, but (according to syllabus) not all, meet the first two; none of them learns about the Mann–Whitney test. However, all of these students do meet the methodology and language associated with hypothesis testing.

	Spearman's rank correlation	Product moment correlation	$\chi^2$ test	Mann–Whitney <i>U</i> -test	Wilcoxon signed-rank test	Sign test	<i>t</i> -tests
Biology	✓		✓				✓
Geography	✓		✓	✓			
Psychology	✓	✓	✓	✓	✓	✓	✓

Table 16 Hypothesis tests in A level Biology, Geography and Psychology

It is thus the case that many of those taking A levels in these three subjects carry out work that covers the full statistics cycle, and this is illustrated in Figure 9.

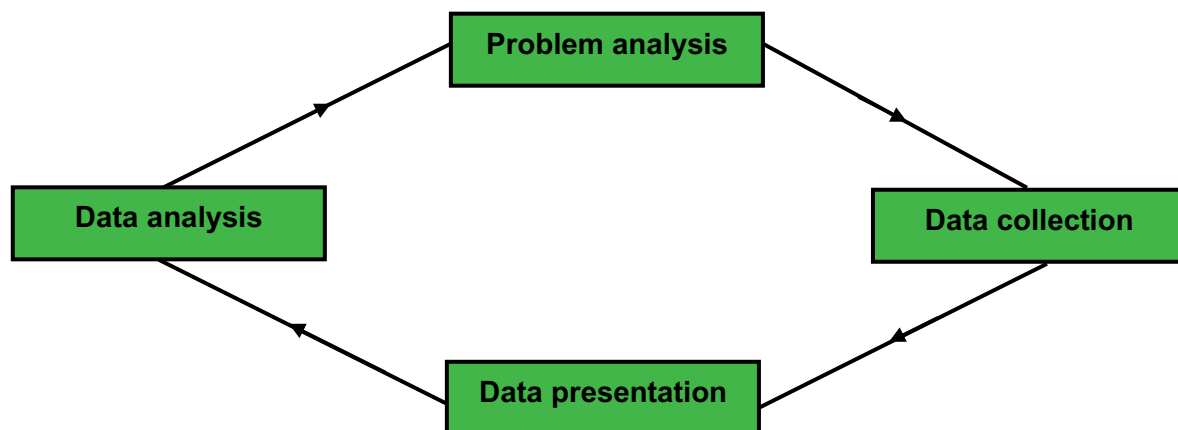


Figure 9 Statistics in A level Biology, Geography and Psychology

There is no hypothesis testing in GCSE Mathematics. At AS level only one of the syllabuses includes it in Statistics 1; in the others it is introduced in Statistics 2. Since a substantial majority of those taking A level Mathematics do not take Statistics 2, they do not meet hypothesis testing within their mathematics. So there is a mismatch between the statistics typically learnt in AS and A level Mathematics and the requirements of these subjects.

#### 7.4.4 Economics and business studies

In economics, A level students do not conduct their own experiments but work quite extensively with secondary data which they are expected to analyse and interpret in terms of the various economic models in the syllabus. Such mathematical models are used extensively at this level (e.g. the economic cycle).

Students are expected to be able to process data into measures which are relevant to these models. Many of these models are designed to explore the consequences of the variability of economic data (e.g. for elasticity). Some of them take the form of curves (e.g. supply and demand curves, aggregate demand curve).

The subject criteria for Business Studies<sup>38</sup> include business analysis and this involves forecasting, data analysis, market analysis, decision making and measures of performance (financial or non-financial). Consequently, according to the way that the analysis is carried out, there is an opportunity for students to follow a procedure that is very like the statistics cycle.

The statistical techniques used in economics and business studies may be put into three categories.

- Many, particularly in economics, are specific to the discipline and so not techniques that would be expected to be covered in mathematics at either GCSE or A level.
- There are, however, a number of generic techniques that are covered within mathematics and are used in these subjects as well.
- Between these two, there is a third group of techniques that are not covered by current mathematics syllabuses, or not to any depth, but probably should be since they are either used in these subjects or in everyday life. Time series and index numbers come within this category.

Table 17 provides an overview of the use of the statistics cycle in these subjects.

Process	Economics	Business Studies
<b>Problem analysis</b>	At A level, little, if any, problem analysis is expected of economics students. They are expected to understand and use standard models and to know which model is appropriate to any situation.	The work needed in business analysis may at times be very close to that involved in problem analysis.
<b>Data collection</b>	At this level, economics students make extensive use of secondary data but do not engage with primary data. So they are neither involved in experimental design nor the subsequent data collection.	There are no requirements for students to collect their own data but market analysis requires them to consider it, and sampling methods are included in some syllabuses.
<b>Data presentation</b>	Students are expected to use a variety of presentation techniques. Some of these are to be found in the statistics in GCSE Mathematics but others are specific to economics.	Students are expected to use a variety of presentation techniques. Some of these are to be found in the statistics in GCSE Mathematics but others are specific to business studies.
<b>Data analysis</b>	Students are expected to analyse secondary data in the context of economic models and to establish algebraic relationships between the variables involved.	Although students are expected to analyse financial data, this is not typically within the context of an investigative cycle.

**Table 17 The statistics used in A level Economics and Business Studies**

#### 7.4.5 Other subjects using statistics

There are several other subjects where students use (or may use) some statistics. Most higher education courses involve some statistics and so it is not surprising that many A level subjects present opportunities for work based on data.

Sociology is a popular A level subject, with over 30,000 candidates a year. Students are expected to be able to display and interpret secondary data, typically using techniques they have learnt in GCSE Mathematics, but they are also expected to know about sampling methods.

**Look at the item below and answer the following questions.**

A research project was conducted into cultural life in Britain. A survey of the UK population looked at how age, class and gender affected taste in mass media and leisure interests.

The survey used a random sample of nearly 1600 people. An additional quota sample of 230 members of British ethnic minority communities was included to make sure that this group was fully represented. This was followed up by household surveys of 44 homes.

Statistical information was gathered on the sample and it was discovered that only 12% of the population attended opera, with a similar percentage attending bingo. Ethnic minorities are less interested in soap operas than the general population.

*Adapted from the ESRC website*

- (a) Using material from the item and elsewhere, explain the meaning of the term **survey**. [10]
- (b) With reference to the item and sociological studies, explain why different forms of sampling are used in social research. [20]

**Example 15 An AS level Sociology question<sup>39</sup>**

Geology is offered by two examination boards. It has a quite small uptake, about 2000 candidates. Students carry out fieldwork which is subject to controlled assessment. They are required to devise their own experiments and to collect their own data. Consequently this work follows part of the statistics cycle; however it often bypasses the more mathematical work described as data analysis.

Environmental Studies is offered by one examination board and has an annual candidature of somewhat under 2000. It is assessed entirely by examination and many of the questions require candidates to interpret displayed data. There is no requirement for students to carry out fieldwork and so to collect their own data.

General Studies is taken by about 40,000 students a year. Among a wide range of other things, candidates are expected to be able to interpret data critically, and this may include an intelligent appraisal of how they were collected.

An increasing number of students (16,000 in 2010 and 25,000 in 2011) are also now doing the *Extended Project* in addition to their A levels. Some projects involve collecting and interpreting data and so include a strong statistical element.

## 7.5 Applied A levels and diplomas

At a strategic level, the provision of academic education through A levels has remained stable for the last 60 years. In matters of detail, there have, of course, been many and at times frequent changes but the overall pattern has remained in place. However, the same is not true of vocational education, which has been the subject of one upheaval after another. There have been several attempts to ensure parity of esteem between vocational and academic routes and this has resulted in quite a number of types of qualifications.

### 7.5.1 Applied A levels

Applied A levels were introduced as replacements for GNVQs. They are available as single awards in the ten subjects: Art & Design, Business, Engineering, Health & Social Care, ICT, Leisure, Media, Performing Arts, Science, Travel & Tourism. The uptake of all of them is small, under 35,000 in total for all ten together. The largest is ICT with 11,000 students in 2011, followed by Business and Health & Social Care, both of which had between 7000 and 8000 students. Most of these courses are also available as double award A levels but the overall uptake is small, fewer than 8000 in total in 2011.

Some Applied A levels include coursework in which students are able to choose the topics they work on. Almost inevitably some of their choices result in the need for data collection, presentation and interpretation.

### 7.5.2 Diplomas

The first diplomas, including those in Engineering and Computing, were introduced in 2008. None of them has had a large uptake and at the time of writing this report it seems likely that few will survive.

The Level 3 Diploma in Engineering includes a compulsory mathematics unit (as well as a much more advanced optional unit). This unit includes some statistics and this is summarised in Table 18. Most of it is in GCSE Mathematics.

<b>Statistics</b>	
The learner will develop knowledge and understand	
1. Data handling	8. Standard deviation
2. Histograms	9. Probability
3. Frequency polygons	10. Expectation
4. Cumulative frequency	11. Dependent event without replacement
5. Arithmetic mean, mode and median	12. Independent event with replacement
6. Percentiles and quartiles	13. Addition law of probability
7. Distribution curves	14. Multiplication law of probability

**Table 18 Engineering diploma: statistics in the compulsory mathematics unit<sup>40</sup>**

This content has been criticised as being unlike that used in the real world of engineering<sup>41</sup>, either on the shop floor or in design. This makes it almost impossible to write good accompanying teaching resources for engineering students. Topics like risk and statistical process control could be introduced with advantage; the importance of both of these to employers is highlighted in ACME's report on Mathematical Needs<sup>42</sup>.

Even if the Diploma in Engineering does not survive, it will be replaced by a similar qualification because there is a need to be filled in training future technicians. In that case, it will be important that the statistics in the new qualification is more appropriate.

## 8 Where in the curriculum is statistics best taught?

### 8.1 The present situation

Figure 10 summarises the statistics that is currently taught within mathematics, as outlined in the previous three sections of this report.

	Primary	11–16	16–19
<b>Problem analysis</b>			
<p>Problem analysis requires modelling and so, according to the problem, can vary from being quite simple to extremely sophisticated and demanding. Although primary school pupils encounter such work, usually in a cross-curricular context, this is not followed up once they go to secondary school or even if they take A level Mathematics; a small number revisit these skills doing A levels in other subjects.</p> <p>At a time when there is considerable criticism of the lack of problem solving skills among young people<sup>43</sup>, this is clearly a wasted opportunity both in terms of their development of life skills and of their understanding of statistics.</p>			
<b>Data collection</b>			
<p>Collecting their own data helps students of all ages to take ownership of their work; it is thus very valuable in helping them to engage with what they are doing and so come to terms with the problems involved, be they statistical or more general. The techniques involved, including sampling and experimental design, can be very sophisticated and the valid use of statistics depends critically upon them. However most students do no data collection in mathematics once they leave primary school and consequently have an incomplete understanding of statistics.</p>			
<b>Data presentation</b>			
<p>All students meet a variety of data presentation techniques throughout the 11 years in which mathematics is a compulsory subject, and some continue to do so up to the age of 18.</p>			
<b>Data analysis</b>			
<p>Data analysis involves the use of a wide variety of mathematical techniques. Inevitably most of this work is beyond the scope of younger students and so it is unsurprising that it is only done to any extent by 16–18-year-olds as part of A level work, be it in mathematics or in other subjects.</p>			

Figure 10 Progression through the statistical processes, by age

Figure 10 illustrates the fact that there are serious discontinuities in the statistics within mathematics in problem analysis and data collection. For a few students the deficits are largely made up in other subjects but for most this is not the case, or only marginally so.

This is clearly an unsatisfactory situation, and one that calls for a systemic solution. In its submission to the National Curriculum Review (2011), the Royal Statistical Society wrote

*Statistics must continue to appear in the mathematics curriculum. It is the Society's belief that it is essential that all learners in these age ranges become acquainted with statistical ideas, indeed it is their right. This is partly to do with general education for ordinary life as a modern citizen. It is partly to do with support of whatever other subjects the learner may be studying. It is partly to do with coherence within the mathematics curriculum itself. It is entirely to do with recognising that the real world exhibits variability, and that risk and chance are everywhere.*<sup>22</sup>

This section of the report examines whether mathematics really is the best place in the school curriculum for statistics. There are three obvious options.

- 1 Statistics teaching is removed from mathematics and carried out within other subjects, using the contexts they provide.
- 2 Statistics is taught as a subject in its own right.
- 3 Statistics continues to be taught within mathematics but steps are taken to overcome the limitations in the current provision.

These options are considered in the remainder of this section. The issues are not the same for the various phases and so for each option separate consideration is given to primary, 11–16 and post-16 education.



## 8.2 Option 1: Statistics is taught outside mathematics in other subjects

### 8.2.1 Primary

The nature of primary school lessons, with one teacher covering all subjects means that there is nothing to be gained by declaring statistics to be part of mathematics or part of other subjects; they are all integrated anyway. However, there are implications, one way or the other, for advice on the time teachers should devote to mathematics.

### 8.2.2 11–16

The 11–16 age group meet statistics in mathematics and in a variety of other subjects. However, the contexts available from other subjects are often not used in mathematics, and there is often little or no coordination of the times when students meet particular topics.

In 2004, the Smith report recommended that at this level statistics should be removed from the mathematics syllabus and that it should be taught in other subjects.

*The Inquiry recommends that ... much of the teaching and learning of Statistics and Data Handling would be better removed from the mathematics timetable and integrated with the teaching and learning of other disciplines (e.g. biology or geography).*<sup>44</sup>

In response to this, the Royal Statistical Society set up a working group which published a position statement entitled *Teaching Statistics Across the 14–19 Curriculum*<sup>45</sup>. It stated *We believe that for the time being statistics should be retained and enhanced within mathematics.*

The main arguments against moving statistics out of mathematics at this level are summarised in Table 19.

Area	Argument
Syllabus coverage	Since each subject uses particular statistical techniques, students' learning would be patchy and fragmented, and dependent on their choice of GCSE subjects.
Ownership	The change would leave no school department with responsibility for statistics and for ensuring that teachers receive appropriate CPD.
Teaching	The statistics knowledge of many teachers of user subjects is limited to the techniques they need to use.

**Table 19 Arguments against moving statistics from the mathematics curriculum**

In a separate development, QCA commissioned the RSSCSE to consider the proposal. The subsequent report made the eleven recommendations given in Table 20. The first of these, that statistics should remain in the mathematics curriculum, was accepted and so the change recommended in the Smith report has not happened.

<b>1. The curriculum</b>	We recommend that the statistics and data handling content of the mathematics curriculum should be seeded through real world examples drawn from science, geography and other subjects but should be retained within the mathematics curriculum.
<b>2. Assessment</b>	We recommend that a range of new ways for assessing the statistics and handling data coursework be trialled. All of these alternatives should be designed to occupy no more than a single week of class time. Of the suggested alternatives in the questionnaires, the in-depth interviews in schools have so far shown that teachers thought the field centre option was impractical. We will also consider other possible approaches.
<b>3. CPD</b>	We recommend that a programme of CPD be developed for Heads of Mathematics with particular regard to teaching how the problem solving approach (data handling cycle) can best be taught. This will also be necessary for Heads of Science and Geography if they are required to teach this material.
<b>4. Teaching materials</b>	We recommend that a comprehensive range of teaching materials be developed and made available to Heads of Mathematics to facilitate the teaching of all topics within the statistics and data handling content of the mathematics curriculum. These should be designed to use real data from other subject areas and should embrace the problem solving approach.
<b>5. Trialling</b>	We recommend that material for trialling in schools be based on the topics: handling data project; measures of location; measures of variation; regression and time series; and inference from data. The chosen topics should be explicitly taught through problem solving.
<b>6. Future strategy for teaching and learning statistics</b>	We recommend that the QCA should promote the teaching and learning of statistics and handling data through the statistical problem solving approach.
<b>7. Resources for teaching and learning statistics</b>	We recommend that the QCA should promote the development of a database of resources containing examples of the use of the statistical problem solving approach which should be made available to school teachers. A common format for the resources, as exemplified by the resources developed and trialled by this project, should be used.
<b>8. Teacher professional development in statistics</b>	We recommend that the QCA should promote the development of online CPD resources designed to demonstrate the use of the resources in Recommendation 7 to school teachers. These should be developed in tandem with the teaching resources.
<b>9. Creating a portfolio of resources</b>	We recommend that the QCA should give priority to the development of online CPD resources that will enable school teachers to take ownership of their CPD needs by facilitating the transformation of their own case studies and examples into further resources in the style of those in Recommendation 7.
<b>10. Development of assessment</b>	We recommend that the assessment of the problem solving approach, described as the Data Handling Cycle in the National Curriculum, should be undertaken by using, and further developing, the approach to this assessment developed by this project.
<b>11. Dissemination</b>	We recommend that a national conference/workshop be organised by the QCA where teachers can learn about the resources and get involved with the creation of further ones in line with recommendations 9 and 10. This could be done under the auspices of the NCETM.

**Table 20 Recommendations of the RSSCSE report to QCA<sup>46</sup>**

### 8.2.3 Post-16

The best possible outcome of taking statistics out of A level Mathematics, and leaving it to be taught in context in the A level courses of other subjects, would be that a student learnt the same amount of statistics as at present. However, many would learn much less.

Those teaching statistics-rich subjects, such as biology, geography and psychology, do not depend on their students learning their statistics in mathematics, which only a minority of them take anyway. Instead they teach the statistics that their students will require. So removing statistics from A level Mathematics would have no effect on lessons in those subjects.

However, many students taking A level Mathematics would no longer learn any statistics, and so would definitely be deprived of opportunities that are open to them at present. Even in the highly unlikely event of a core of statistics being written into the A level syllabuses of subjects other than mathematics, it would not cover the material in the more advanced modules in A level Mathematics and Further Mathematics; that would be completely lost.

Moving statistics out of mathematics and into other subjects would produce no winners and plenty of losers among this age group.

## **8.3 Option 2: Statistics is taught as a subject in its own right outside mathematics**

### **8.3.1 Primary**

The nature of primary school lessons, with one teacher covering all subjects, means that there is nothing to be gained by declaring statistics to be a separate subject.

### **8.3.2 11–16**

As described in Section 6.2, statistics already exists as a separate GCSE subject; it is taken by about 7% of the age cohort. So the consequence of replacing the statistics within mathematics, which is taken by everyone, by a separate subject, would almost certainly be a massive reduction in the amount of statistics learnt by this age group.

### **8.3.3 Post-16**

The likelihood that mathematics, in some form, will become compulsory for all students in this age group raises possibilities for statistics that merit serious consideration.

The requirement could be expressed in a way that ensures that all level 3 students take a core of statistics covering much of the content in the present Statistics 1 modules and some of Statistics 2 modules, and investigative work covering the whole statistics cycle. This could remove such work from mathematics syllabuses while leaving the more mathematical aspects of the subject in place.

Such a change will not happen in the short term, and so there is time to consider its merits, and any unwanted side effects. It would, for example, be important to ensure that any new arrangement does not deter students from doing A level Mathematics.

## **8.4 Option 3: Statistics continues to be taught as part of mathematics**

### **8.4.1 Primary**

Although there are issues to be resolved regarding the amount of statistics taught in primary schools, and the time allocated to it, there is no obvious advantage to be gained by changing the present arrangement of treating it as part of the mathematics curriculum.

### **8.4.2 11–16**

There has been serious consideration of the merits of placing statistics in the mathematics curriculum or outside it. While there are problems with the current provision, it would seem inevitable that other, and more serious, problems would occur if statistics were to be moved to another part of the school curriculum.

### **8.4.3 Post-16**

Under the present arrangements, statistics is effectively taught separately in mathematics and in those subjects that use it. While this arrangement seems bizarre, it is a reason for a better coordinated provision rather than for removing it from within mathematics.

## **8.5 Conclusions about the current provision**

The current situation is a product of our national education system and the options within it. It is the conclusion of this report that while this system remains in place, statistics will continue to be best placed within mathematics, with students' learning reinforced in other subjects. This conclusion should not be taken as support for the detail of the present provision; there is much scope for improvement and changes are proposed in Section 9.

There are, however, dangers for statistics in this course of action. Not all mathematicians value statistics and there is the consequent danger that the time allocated to it will be under pressure. This happened in 2003–4 when the content of A level Mathematics was being rewritten in the wake of Curriculum 2000. There were no cuts to the pure mathematics; instead they fell entirely on the applied mathematics, with one complete module, typically statistics or mechanics, being removed. There is also the danger that the particular needs of statistics, both as a practical and as a cross-curricular subject, will not be acknowledged.

Whenever, as seems possible in the case of mathematics post-16 in the medium term, there are changes to the balance between compulsory and optional elements in the overall curriculum, this conclusion will need to be reviewed.

## 9 Improving the statistics provision

The conclusion of the previous section of this report was that the best place for statistics in the school curriculum is within mathematics but that, for this to work well, changes are needed. In this section, suggestions are made as to what those changes should be.

### 9.1 Problem analysis and data collection

The unsatisfactory, and often non-existent, provision in problem analysis and data collection has already been highlighted.

Although many primary schools do work that involves these processes, doing so is a matter of good practice rather than a requirement of the National Curriculum. The data handling cycle is not in the programme of study for Key Stage 2; it comes in at Key Stage 3.

The situation at GCSE is manifestly unsatisfactory, with the data handling cycle written into the Key Stage 4 programmes of study, and into GCSE syllabuses, but not effectively assessed.

At AS and A level there are two different but related problems.

- The syllabuses for the Statistics 1 module in some syllabuses do not include hypothesis testing and so do not support the statistical work that is being done in those other subjects that make extensive use of statistics.
- While almost all students who take AS or A level Mathematics do at least one statistics module, only a small minority of them are given any experience of problem analysis and data collection.

## 9.2 Content (up to GCSE)

The statistics content in the National Curriculum is not substantial. The pattern is better described as repeating the same topics year after year than as a steady introduction of new ideas. There is space for more topics to be covered at both primary school and in Key Stages 3 and 4.

A number of topics are commonly used in everyday life or in other subjects at this level, but are not covered in GCSE Mathematics. There are also topics that are widely used, and misused, and so would benefit from being given more attention in the work leading up to GCSE Mathematics. When a topic is used in other subjects or in everyday life, it is reasonable to expect good practice to spread out from the mathematics classroom, including an awareness of any pitfalls. This does not always happen.

### 9.2.1 Index numbers

Index numbers, and other ratio-based indices, are used in many other subjects. There is a body of associated theory, involving weighted means and adaptation to changing circumstances, that could well be taught within mathematics, for example at GCSE, but is not. So the only students who meet the principles on which a measure like the RPI is calculated are those who take economics. At the time of writing this report, a pilot is being conducted of twin GCSEs; index numbers would fit particularly well in the syllabus called *Applications of Mathematics*.

### 9.2.2 Time series

Time series are widely used outside mathematics. Moving averages are covered in the Higher tier of GCSE Mathematics but not in the Foundation tier, so many students miss out on the associated theory, involving trend, seasonal and other cyclic variation as well as random variation.

### 9.2.3 Errors

In the physical sciences, the variability of data is often largely a result of experimental error, typically resulting from inaccurate measurement; the same is also true in the workplace. The relationship between the accuracy of information and the appropriate number of figures to which it is given does fall within GCSE mathematics, but more could be done on the propagation of errors and their graphical representation, for example by error bars.

### 9.2.4 Description of variables

Another area that is not well covered in mathematics is the description of variables. Many important ideas come under this heading, for example:

- different types of scales,
- random and non-random variables,
- dependent and independent variables,
- repeated measures.

It would not be unfair to say that much of this is brushed under the carpet in mathematics syllabuses, both GCSE and A level, but the distinctions involved are really important for anyone actually using statistics, whether in support of another subject or in employment.

### 9.2.5 Risk

An understanding of risk is important as part of everyday life, in employment and in experimental work. It is a topic in the syllabus of *Applications of Mathematics*, one of the twin GCSEs currently being piloted. This is the first time that it has featured in any mathematics syllabus at this level and it is requiring new teaching and assessment materials. It is clearly desirable that this early work on risk should become embedded in GCSE Mathematics.

### 9.2.6 Bivariate data

Although pupils meet bivariate data at an early age, the overall coverage to which they are exposed is often patchy.

Scatter diagrams are covered in Level 6 of the National Curriculum. However, they are actually taught in many primary schools and so could realistically be added to the Key Stage 2 curriculum. It is common for students to be told to add a line of best fit, drawn by eye, to a scatter diagram but more attention needs to be given to cases where it is inappropriate to do so.

The ideas of correlation and causation, often linked together, are widely used outside mathematics. More could be done within mathematics to emphasise the danger of assuming that correlation implies causation.

### 9.2.7 The Normal distribution

The Normal distribution is not covered in GCSE Mathematics but given its importance as a distribution of naturally occurring variables (lengths, times, etc.), it could be a Higher tier topic. This would inform its possible use in subjects like biology and geography.



## 9.3 Content (AS and A level)

Locating statistics in the mathematics curriculum imposes obligations on those responsible for designing its content and assessment, particularly for the first statistics module in AS Mathematics, Statistics 1. This is taken by a large number of students; it will be the only statistics module that many of them take.

Statistics 1 serves two quite different purposes.

- It provides learning and support for those using statistics in other subjects.
- It is the first in a sequence of modules leading ultimately into academic statistics in higher education.

The existing Statistics 1 syllabuses were subject to another constraint when they were drawn up in 2003. At that time GCSE Mathematics had three tiers of entry, Higher, Intermediate and Foundation. Students taking the Intermediate tier could obtain a grade B in mathematics and so expect to proceed to AS and A level. This meant that the content of what was then Higher tier GCSE could not be assumed knowledge. So some topics, largely data presentation, had to feature in Statistics 1 syllabuses.

Changing the status of such topics, so that they become assumed knowledge, rather than part of the Statistics 1 syllabuses, will free up space for new topics and ideas.

### 9.3.1 Hypothesis testing

As already described, hypothesis testing is carried out in a number of other subjects. So, if it is to fulfil its role in supporting these subjects, Statistics 1 should include hypothesis testing.

The tests used in biology, geography and psychology are listed in Table 16 (see page 47). It will not be possible to cover all of them in Statistics 1. What can, and should, be covered is the essential methodology and associated language: null and alternative hypotheses, significance level, one- and two-tailed tests, test statistic, critical value,  $p$ -value, etc. For students who have learnt one type of hypothesis test, other tests will present few problems.

### 9.3.2 Standard deviation

Students moving from GCSE to AS level Mathematics meet standard deviation for the first time. Although both the British Standard and the International Standard<sup>47</sup> specify the use of divisor  $n-1$  when this is calculated from sample data, only some of the syllabuses require, or even encourage this. Others expect the divisor  $n$ . This situation needs to be resolved.

All of biology, geography and psychology require a knowledge of standard deviation; in some cases this is stated explicitly and in others it is implicit in the particular hypothesis tests that students are expected to know. Teachers of all subjects should be able to look to mathematics for correct practice.

### 9.3.3 Correlation and regression

The idea of correlation is used in many subjects, with rank correlation more common than product moment correlation. There is, however, considerable scope for misunderstandings, leading to invalid work and conclusions. It is important that best practice is established in mathematics. This includes the use of correlation coefficients as test statistics so that students learn how their interpretation is related to sample size.

In mathematics syllabuses, correlation is usually presented alongside linear regression. This can lead to misunderstanding of the different types of variables involved.

## 9.4 Special techniques

While much statistics is taught within mathematics, or reasonably could be, this cannot always be the case. Many subjects have some of their own techniques for collecting, presenting and analysing data. It is unrealistic to expect all of these to be covered within mathematics syllabuses, and so such special techniques should be taught primarily within the subjects where they are used. They may provide enrichment for statistics lessons within mathematics, while not being part of the assessed syllabus.

Examples include: mark–release–recapture (biology); population structures (geography); use of game theory such as the prisoner’s dilemma (economics); statistical definition of abnormality (psychology).

Similarly, mathematics has its own special techniques. Those who take the higher statistics modules as part of Further Mathematics begin to meet the mathematical theory underpinning the subject with topics, like estimators and generating functions, which will not have applications in the A level courses of other subjects.

## 9.5 New courses post-16

It is very likely that new post-16 courses will soon be developed; the target students will be those who currently give up mathematics after GCSE.

These new courses will almost certainly contain substantial amounts of statistics, but some will certainly also include other mathematics as well. Consequently it would seem, at this pre-development stage, best to see them as part of an extended mathematics provision rather than being outside it.

These courses will be designed for those who currently do no mathematics or statistics beyond GCSE. Since many such students are currently very glad to have given up all forms of mathematics, the new courses will only be successful if they succeed in engaging the interest and enthusiasm of this clientele. They will require careful design and the statistical content, and the way that it is presented, will be critical in this.

In Section 7.5, the point was made that the statistics currently taught on vocational courses is out of line with that commonly used in today's workplace. Suggestions for more relevant content are included in ACME's Mathematical Needs report<sup>48</sup>. It will be important for all the new courses, and not just those designed for vocational students, that the statistics content is up to date and relevant to the next stages of their lives, whether in the workplace or in higher education.

These courses clearly need a strong and creative input from practising statisticians as well as from those with education expertise.

## 10 Where should ownership of statistics at this level lie?

While this report was in a draft stage, the Secretary of State for Education gave a speech at the Royal Society in which he emphasised the importance of mathematics and statistics. Michael Gove said:

*And what about statistics? There are a vast array of issues that people are confronted with in daily life – from health scares to claims about the effect of drugs to financial news – which require statistical understanding. But studies have repeatedly shown how poor our collective understanding of such issues is. In its present form, GCSE Maths does not enable children to understand conditional probability, Normal distributions or randomness. Should this be something we should look to change?<sup>49</sup>*

The development of a better statistics provision in the coming years is clearly a real possibility, but it will need to be well designed and led.

This report recommends that at school and college level, statistics should continue to lie within the mathematics curriculum. However, as has already been explained, such a policy is not without risk.

There are those who do not acknowledge the importance of statistics and who resent time being given to it at the expense of other aspects of mathematics that they regard as more important. There is always the risk of the statistics taught in schools and colleges being moulded to become more of an exercise in pure mathematics. Another ever-present danger is that those who do not understand the value of statistics exert political pressure for a reduction in the extent to which it is taught: ‘More algebra and less data handling’ is a beguiling message.

So, although at an operational level, statistics is well placed within mathematics departments, with an institution’s head of mathematics responsible for overseeing its teaching, this does make it vulnerable at the strategic level.

The question of who is responsible for statistics policy needs to be asked and answered. Who is going to protect its position? Who is going to tell those responsible for the rest of the mathematics curriculum what elements of statistics should appear in it, and where?

This is not a trivial matter. As a crucially important discipline in its own right, statistics needs to be professionally and consistently represented on all relevant decision making bodies. The Royal Statistical Society is well placed to take a lead in this.

The concerns expressed above may read rather like a statement of conflict between statistics and mathematics. That is not, of course, how the relationship should be. Statistics is itself a branch of mathematics, albeit with its own special needs; it is supported by and supports many other aspects of mathematics. The recommendation that statistics be properly represented alongside the rest of mathematics at policy level is designed to ensure that both disciplines benefit from their close association.

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## Opportunities for using statistics across the GCSE curriculum

Subject	Assessment requirements	Teaching and learning opportunities
<b>Art and Design</b>	None. Although there is 60% Controlled Assessment, there is no requirement for work of a statistical nature.	Students are expected to show evidence of research in their portfolio of work. According to their choice of theme, this allows the possibility of some statistical work in using a survey to determine, for example, the factors people regard as important in a particular design.
<b>Biology</b>	Practical work is the subject of Controlled Assessment. In Biology, the work involves sampling, data collection and interpretation. It contributes 25% to the overall assessment.	Natural variability is part of the fabric of biology and so statistical methods are relevant to almost all aspects of the subject. Particularly rich opportunities arise through field and practical work: designing experiments, sampling and data collection leading to data display and interpretation.
<b>Business Studies</b>	Business Studies candidates are required to carry out some research, including data collection and presentation; there is 25% Controlled Assessment. Also, some of the examination questions require interpretation of data.	There are many opportunities for the use of statistics in business studies. Students are expected to interpret graphs, charts and data about finance and to perform break-even analysis, with obvious connections to statistics. While the use of secondary data fits naturally into many aspects of this subject, market research provides particularly rich opportunities for the use of the statistics cycle.
<b>Chemistry</b>	Practical work is the subject of Controlled Assessment. In Chemistry, the work involves data collection and interpretation. It contributes 25% to the overall assessment.	Practical work in chemistry often gives rise to data. However, sampling is rarely involved. The same is true of experimental design; most experiments in chemistry at this level use standard techniques. The variability in the data is usually due to experimental error but this still presents opportunities for statistical work; suitable techniques can be used to estimate it and to display it on diagrams. In addition, chemistry students have many opportunities to use the data presentation techniques they have learnt in mathematics.
<b>Citizenship Studies</b>	None. Although there is 60% Controlled Assessment, there is no requirement for work of a statistical nature.	Two aspects of citizenship present particularly rich opportunities. One of these is the idea of risk, requiring understanding and interpretation of probability. The other is understanding information and data from a variety of sources, including advertising and the media.
<b>Design and Technology</b>	None. Although there is 60% Controlled Assessment, there is no requirement for work of a statistical nature.	The properties of materials provide opportunities for students to learn about natural variability. In real life, understanding customers' taste is an important consideration in product design; research into this provides opportunities for statistical work.

<b>Economics</b>	The assessment is entirely by examination. Some of the questions require candidates to interpret and display data. At times this is done in the context of particular economic models (e.g. elasticity), representations (e.g. supply-demand curves) and indices (e.g. RPI).	Economics offers many opportunities for data presentation and interpretation. Many of these involve the application of statistical techniques that have been covered in GCSE Mathematics in the context of models that are specific to economics. In addition, economics students make extensive use of secondary data and there are considerable opportunities for work that explores the sampling methods involved in using large data sets.
<b>Geography</b>	The subject criteria for Geography require fieldwork and out of class learning. This is the subject of Controlled Assessment, which contributes 25% overall.	There are many opportunities for the use of statistics in geography, including sampling and data collection, data display and interpretation. Many students experience these through their fieldwork. In addition, it is common for teachers to introduce students to techniques, such as Spearman's rank correlation, that are beyond the strict requirements of the GCSE syllabuses.
<b>History</b>	None. Although there is 25% Controlled Assessment, there is no requirement for work of a statistical nature.	Many aspects of history are rich in data, particularly the social and economic parts of the subject. Students need to be able to extract information from source material and at times this involves data collection, display and interpretation. The Controlled Assessment provides a particular opportunity for data rich work to be covered in class.
<b>Home Economics</b>	None. Although there is 60% Controlled Assessment, there is no requirement for work of a statistical nature.	The section entitled Diet and Health offers opportunities for the collection and display of data relating to healthy diets and the different dietary needs of family members. Opportunities in the section on child development involve the use of time series and developing an understanding of natural variability.
<b>ICT</b>	None. Although there is 60% Controlled Assessment, there is no requirement for work of a statistical nature.	There are many opportunities for students to learn how to process and display data using their own methods and existing packages.

<b>Leisure and Tourism</b>	There is 60% Controlled Assessment in Leisure and Tourism, based upon research by the candidate. This may involve interviews, surveys, observation or questionnaires.	<p>The subject criteria require students to carry out tasks in which they analyse issues and problems; identify, gather and record relevant information and evidence; analyse and evaluate evidence; make reasoned judgements and present conclusions.</p> <p>Clearly there are opportunities for students to undertake statistical work in the course of their research. This can, for example, involve information relating to industry trends over a period of time. The data may be from a range of sources and be presented in different ways. Students are expected to present their data appropriately in support of the analysis in their work. Such work may be practised as part of the teaching on a variety of topics ahead of the research that will actually form part of the assessment.</p>
<b>Media Studies</b>	There is 60% Controlled Assessment and this may be based on research by the candidate into media products.	The research aspects of Media Studies (often into audiences and institutions) require primary and secondary research that involves collating, displaying and interpreting raw data from secondary source materials or primary research sources. There are thus significant opportunities for students learning this subject to undertake statistical work.
<b>Physical Education</b>	There is 60% controlled Assessment in Physical Education. In one element of this, candidates are expected to carry out research into lifestyle and into performance. This involves data collection, presentation and interpretation.	There are clear opportunities for physical education students to undertake statistical work while learning the subject. The subject criteria specify the need for them to develop their ability to analyse and evaluate performance and to identify key priorities for improvement. They have the opportunity to measure subjects against normative statistical data, for example their health (e.g. BMI) and fitness (e.g. sit and reach tests), and analyse the results. They also have the opportunity to present their assessment evidence using (basic) statistical methods and they are encouraged to think about how their observations are measurable.

<b>Physics</b>	Practical work is the subject of Controlled Assessment. In Physics, the work involves data collection and interpretation. It contributes 25% to the overall assessment.	Practical work in physics often gives rise to data. However, sampling is rarely involved and often there is little scope for experimental design. The variability in the data is usually due to experimental error but this still presents opportunities for statistical work; suitable techniques can be used to estimate it and to display it on diagrams. In addition, physics students have many opportunities to use the data presentation techniques they have learnt in mathematics. There is an opportunity for creative teachers to explore the similarity between the scientific methodology underpinning practical work in physics and the statistics cycle.  In addition, physics includes work on radioactivity, and suitable simulations can provide students with first-hand experience of random processes.
<b>Psychology</b>	The assessment is entirely by examination. Candidates are required to answer questions about planning, conducting and interpreting research in a written paper. This involves processes within the statistics cycle: experimental design, data collection, presentation and interpretation.	Students are not required to provide evidence of having carried out any research for themselves but those who have not done so will clearly be at a disadvantage. There are thus opportunities for psychology students to use the techniques they have learnt in mathematics, to experience how they fit into an investigative cycle and to understand how this leads on to some important ideas that are not taught in mathematics: e.g. null and alternative hypotheses, sampling methods, types of variables, reliability, bias.
<b>Sociology</b>	The assessment is entirely by examination. Candidates are expected to answer questions about specific examples of research which they have read before the examination. This is likely to cover sampling, data collection, presentation and interpretation.	Students are not required to provide evidence of having carried out any research themselves but clearly will be at a disadvantage if they have not done so. Much of the work in sociology involves the use of secondary data; there is a clear need to obtain samples that are representative of the various groups involved. There are thus many opportunities for experimental design as well as the subsequent data presentation and interpretation.

**Table 21 Opportunities for using statistics across the GCSE curriculum**

**Notes**

- A number of GCSE subjects are not included in this table. There are two reasons for these omissions.
  - In the case of Classics, Drama, English (Language and Literature), all the Modern Foreign Languages, Music, Performing Arts and Religious Studies, there is neither a requirement for statistics to be used nor realistic opportunities for it to be used during teaching.
  - The number of subjects available at GCSE is very large. Many of these have low uptake; some are not offered by all the examination boards. Since listing them all would have made the table hard to read and interpret, only the main subjects (those with higher uptake) have been included.
- Science is covered by the single subjects, Physics, Chemistry and Biology.



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