Is the Carli index flawed? Assessing the case for the new RPIJ

Peter Levell

Institute for Fiscal Studies

November 10th 2016
Historically, the UK has had two main measures of consumer price inflation: the CPI, and the RPI.

In March 2013, the ONS started to publish a new inflation index - the RPIJ.

UK Statistics Authority announced it would no longer recognise the RPI as a ‘national statistic’.

However the RPI is still published and used for:
- uprating excise duties
- government gilts
- price caps of regulated industries etc.
RPI, CPI, RPIJ (2000-2016)

Inflation rate %
CPI  RPI  RPIJ

Is the Carli index flawed?
November 10th 2016
RPI and CPI

- Two primary differences
  - Coverage: RPI includes owner occupied housing costs
  - Formula effect: RPI and CPI use different formulae at ‘elementary level’
- In particular, the CPI does not use the Carli index
- The RPIJ replaces the Carli index with the Jevons as is used in the CPI
  - Follows recommendation of Diewert (2012)
Is the Carli index flawed?

Discuss and contribute to the three approaches to selecting the appropriate index numbers

1. The *test* approach
2. The *economic* approach
3. The *statistical* approach

No consensus on which of these is most important!

Find that none of these offers clear support for the Carli index
Some preliminaries
Aggregation (stylised example)

Overall index
- CPI

Group level
- Other classes (e.g., Food)
- Alcoholic beverages
  - Beer
  - Wine
    - Red wine
    - White Wine
      - Red wine in certain shop type in SE of England
      - Red wine in other shop types/regions

Class level
- Beer
- Wine

Item level
- Red wine
- White Wine

Elementary level
- Red wine in certain shop type in SE of England
- Red wine in other shop types/regions
Aggregation: what is an elementary aggregate?

- While at most stages the ingredients of each index are weighted according to relative quantities purchased...
  - For example we might use the relative budget shares of white wine and red wine
- ....at the elementary level, the ONS *does not have expenditure information*
  - It is much harder to know how much is spent on particular brands of red wine relative to others
- Elementary indices must therefore be unweighted averages of various kinds
Some preliminaries

Elementary indices

1. The Carli index: \( P_C (p_0, p_1) = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{p_1^i}{p_0^i} \right) \)

2. The Dutot index: \( P_D (p_0, p_1) = \frac{1}{N} \sum_{i=1}^{N} \frac{p_1^i}{p_0^i} \)

3. The Jevons index: \( P_J (p_0, p_1) = \prod_{i=1}^{N} \left( \frac{p_1^i}{p_0^i} \right)^{\frac{1}{N}} = \frac{\prod_{i=1}^{N} (p_1^i)^{\frac{1}{N}}}{\prod_{i=1}^{N} (p_0^i)^{\frac{1}{N}}} \)

- Important result: The Jevons is \emph{always} less than or equal to the Carli (Jensen’s inequality)
The test approach
The test approach

- Does the index satisfy common-sense axioms?

1. **Positivity:** $P(p_0, p_1) > 0$
2. **Identity:** $P(p, p) = 1$
3. ...

Positivity: $P(p_0, p_1) > 0$
Identity: $P(p, p) = 1$
Time reversal

- The Carli fails some important properties

\[ P(p_0, p_1) P(p_1, p_0) = 1 \]

- In fact: \( P_C(p_0, p_1) P_C(p_1, p_0) \geq 1 \)
- Thus the Carli is ‘biased’ upwards
‘Price bouncing’ example

<table>
<thead>
<tr>
<th>Prices</th>
<th>Period 0</th>
<th>Period 1</th>
<th>Period 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shop A</td>
<td>1</td>
<td>1.25</td>
<td>1</td>
</tr>
<tr>
<td>Shop B</td>
<td>1.25</td>
<td>1</td>
<td>1.25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Period (0,1)</th>
<th>Period (1,2)</th>
<th>Chained</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carli</td>
<td>...</td>
<td>(1.25 + 0.8)/2 = 1.025</td>
<td>1.025</td>
</tr>
<tr>
<td>Dutot</td>
<td>...</td>
<td>1.125/1.125 = 1</td>
<td>1</td>
</tr>
<tr>
<td>Jevons</td>
<td>...</td>
<td>√1.25 × 0.8 = 1</td>
<td>1</td>
</tr>
</tbody>
</table>
The test approach: a summary

- The Carli fails the important time reversal test with an upward bias
  - As a result also fails a stronger price bouncing test
- This failure underlay recommendation of Diewert (2012) to end the use of the Carli index in the RPI
How serious is the failure of time reversal?

- Even in an index using the Jevons at an elementary level, the overall index is not time-reversible
  - however it will be *less* sensitive to biases than those constructed using a non-time-reversible index
The economic approach
The economic approach

- Does the index do a good job of approximating the cost of living?
  - How much must we increase consumers’ incomes to fully compensate them for a price change?

- This approach ideally would explicitly incorporate consumers’ substitution responses in response to price changes
  - if price of one good rises relatively to another, consumers might take advantage by substituting away from that good

- Measured by a Cost of Living Index (COLI)
The economic approach

- Two possible COLIs
  - Laspeyres: $P_L (p_0, p_1) = \frac{\sum p_1^i q_0^i}{\sum p_0^i q_0^i} = \sum w_0^i \left(\frac{p_1^i}{p_0^i}\right)$
  - Geometric Laspeyres: $P_{GL} (p_0, p_1) = \prod_{i=1}^{N} \left(\frac{p_1^i}{p_0^i}\right)^{w_0^i}$

  where $w_0^i$ corresponds to the budget share of good $i$ in period 0

  Correspond to Leontief ("no substitution") and Cobb-Douglas ("constant shares") preferences respectively

- Not unreasonable to suppose some substitution at the elementary level
The economic approach at the elementary level (1)

Diewert (2012) writes

'...the economic approach cannot be applied at the elementary level unless price and quantity information are both available'

Without budget share information we cannot say that

- the Carli approximates the Laspeyres or that
- the Jevons approximates the Geometric Laspeyres

\[ \Rightarrow \text{ we should use the test or statistical approaches at this level instead} \]
However, if we want to apply the economic approach consistently at different levels of aggregation then this is not a very satisfying solution!

Propose instead a constructive principle for selecting appropriate index numbers without quantity information...
Principle of maximum entropy (1)

- Imagine we have a weighted die with a known number of sides but where the probabilities of dice rolls are unknown.
  - What can we say about the probability of each individual role?
- Laplace’s ‘principle of insufficient reason’ provides a first step.
  - You should set the probabilities to be equal unless you have reason to do otherwise.
Principle of maximum entropy (2)

- The PME (Jaynes, 1957a,b) combines Laplace’s principle with any information we do have to obtain a prior probability distribution.
- An objective function that achieves this is Shannon’s entropy function

$$H(p) = -p \ln(p)$$

- Maximise this subject to what we do know (e.g. the average dice roll)
PME favours the Jevons (1)

- Apply the PME to vectors of budget shares in both periods
  \[
  \max_{w_0, w_1} H(w_0, w_1) = -\sum_t w_t' \ln w_t \quad \text{subject to} \quad \sum_i w_t^i = 1 \quad \text{for} \quad t = 0, 1
  \]

- This will set shares equal across goods in both current and base periods: \( w_t^i = 1/N \) for all \( i \) and \( t = 0, 1 \)

- Implies both that the Geometric Laspeyres is the target index and that the Jevons approximates this
  \[\implies \text{favours the Jevons index} \]
PME favours the Jevons (2)

- An additional restriction we can impose is that consumers decisions are governed by restrictions of rational choice.
- These are represented by the Generalised Axiom of Revealed Preference (GARP)

\[
\max_{w_0, w_1} - \sum_t w'_t \ln w_t \\
\text{subject to } \{p_0, p_1; w_0, w_1\} \text{ satisfies GARP and } \sum_i w^i_t = 1 \text{ for } t = 0, 1
\]

- It turns out that the solution to this problem is the same: \( w^i_t = 1/N \) for all \( i \) and \( t = 0, 1 \).
Summing up

- Statistical approach (not discussed) is context dependent
  - Carli and Jevons differ in bias and variance properties
- Test approach favours the Jevons over the Carli
- Economic approach favours the Jevons over the Carli
- Hence I concur with the decisions of ONS and United Kingdom Statistics Authority
