constructed a spatiotemporal model involving spatial, temporal and seasonal effects and interactions.

Fig. 8(a) plots the spatial locations of the observations within a specific time window, using colour to indicate the value of the pollution level of each observation. This is an interactive plot which enables the time evolution of the pollution measurements to be explored in a more effective manner than the simultaneous viewing of a set of static plots for selected time windows. The animation employs a time window whose width is indicated in the horizontal bar. The shading that is shown here indicates that the time window is in fact created by a filter, or weight, function, which allows observations to move smoothly into and out of the plotted data. This is achieved by using the hue saturation value form of colour representation (see Manjunath *et al.* (2001)) and downweighting the saturation component according to its distance from the current centre of the time window. The effect of these operations is to create a smooth transition as observations enter and leave the plotted data.

Figs 8(c) and 8(d) show how the time patterns at specific spatial locations can also be explored, where a click on Fig. 8(c) identifies a spatial region within which the pollution values are plotted over time and which may then be dragged across the plotting area. This is a form of interaction with plots known as ‘brushing’ (Becker and Cleveland, 1987) which has been adapted here to the spatiotemporal setting.

Bowman *et al.* (2009) proposed a flexible regression model for log-SO_2, y, with terms involving spatial location, s (two dimensional), time in years, t, and month, z, the last to reflect the seasonal signal. In standard model notation, this can be expressed as

$$y=\mu+m_s(s)+m_t(t)+m_z(z)+m_s(s):m_t(t)+m_s(s):m_z(z)+m_t(t):m_z(z)+\varepsilon,$$

where m denotes a smooth function, ‘:’ denotes interaction terms and ε is an error term. This model was fitted by Bowman *et al.* (2009) through local linear regression and the backfitting algorithm. Here a p-spline representation of each smooth function is used, as described by Eilers and Marx (1996), with 6 and 12 degrees of freedom for one- and two-dimensional terms respectively. The behaviour of the error term ε is modelled by a separable combination of a spherical covariance function $\exp\{-(d_s/\nu)^2\}$ of spatial distance d_s and temporal correlation of auto-regressive AR(1) form on a monthly scale, with correlation parameter ρ. For convenience, the estimated values of $\hat{\nu}=0.098$ and $\hat{\rho}=0.569$ that were reported by Bowman *et al.* (2009) are used. After estimation of model terms by penalized likelihood based on independent errors, with estimated standard deviation 0.793, an estimated covariance matrix can then be used to construct adjusted standard errors. Bowman *et al.* (2009) give the details.

Fig. 8(b) shows the interaction of the spatial and seasonal terms $m_s(s):m_z(z)$. These are the adjustments to an additive model that are required to describe the SO_2 patterns effectively. (This is a case where controls to display the patterns at particular positions are very helpful.) To highlight the need for these adjustments, contours corresponding to 2 or more standard errors from 0 draw attention to the areas where the evidence for interaction is strong. The animation goes on to display the main effects and interaction together: $\mu+m_s(s)+m_z(z)+m_s(s):m_z(z)$. Here the plot is dominated by the main effects but the contours remain to highlight the presence of the interaction term. This is an example of graphical display involving not only data but also a sophisticated model which can provide clear insight into a complex environmental process.

Fig. 8 was created through the `rp.spacetime` function in the `rpanel` package (Bowman *et al.*, 2007) for `R` (R Development Core Team, 2013). Jones *et al.* (2014) described software which creates spatiotemporal animations in a convenient automatic manner, specifically designed for the context of groundwater monitoring.

6. Discussion

The graphics that are discussed in the paper aim to provide displays of uncertainty which are intuitive, particularly for a non-technical audience, but which are aligned as closely as possible with the technical construction of the underlying inferential methods. One underlying theme has been the use of colour intensity shading to provide graphics which are more consistent with the fuzzy nature of uncertainty and which counteract the 'inside–outside' interpretation of confidence intervals, building on the work of Jackson (2008). A second theme has been the use of animation which, in particular, enables graphics to remain in the same visual space as the data and model of interest.

Colour selection is an important general issue as this has major implications for the perception of changes across categories or along continuous scales. This is a broad topic which was very helpfully discussed by Zeileis *et al.* (2009).

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