Bayesian Semi-Parametric Analysis of Semi-Competing Risks Data

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Pancreatic Cancer

- Approximately 56,000 individuals will be diagnosed with pancreatic cancer in the U.S. this year
- Unfortunately there are no effective screening modalities
  - patients are usually diagnosed at late stages
- Large majority are not eligible for surgical treatment
  - chemotherapy is administered in the context of palliative care
- Prognosis is very poor
  - 5-year survival rate is 9%

Ongoing collaboration

- Broad goal is to characterize and understand variation in the quality of end-of-life care for patients diagnosed with pancreatic cancer
- Quality can be measured in many ways
- Our immediate focus is on readmission
  * readmission after discharge from the hospitalization at which the diagnosis was given
- Hospital-specific readmission rates are calculated and reported by CMS
  * determine, in part, a hospitals reimbursement rate for the subsequent year
  * logistic regression: used for health conditions with effective treatment options and low mortality

Challenges

Death as a competing risk

- Consider outcomes among $N = 16,051$ Medicare patients:
  - between 2005-2008
  - inpatient care claims, including hospitalizations

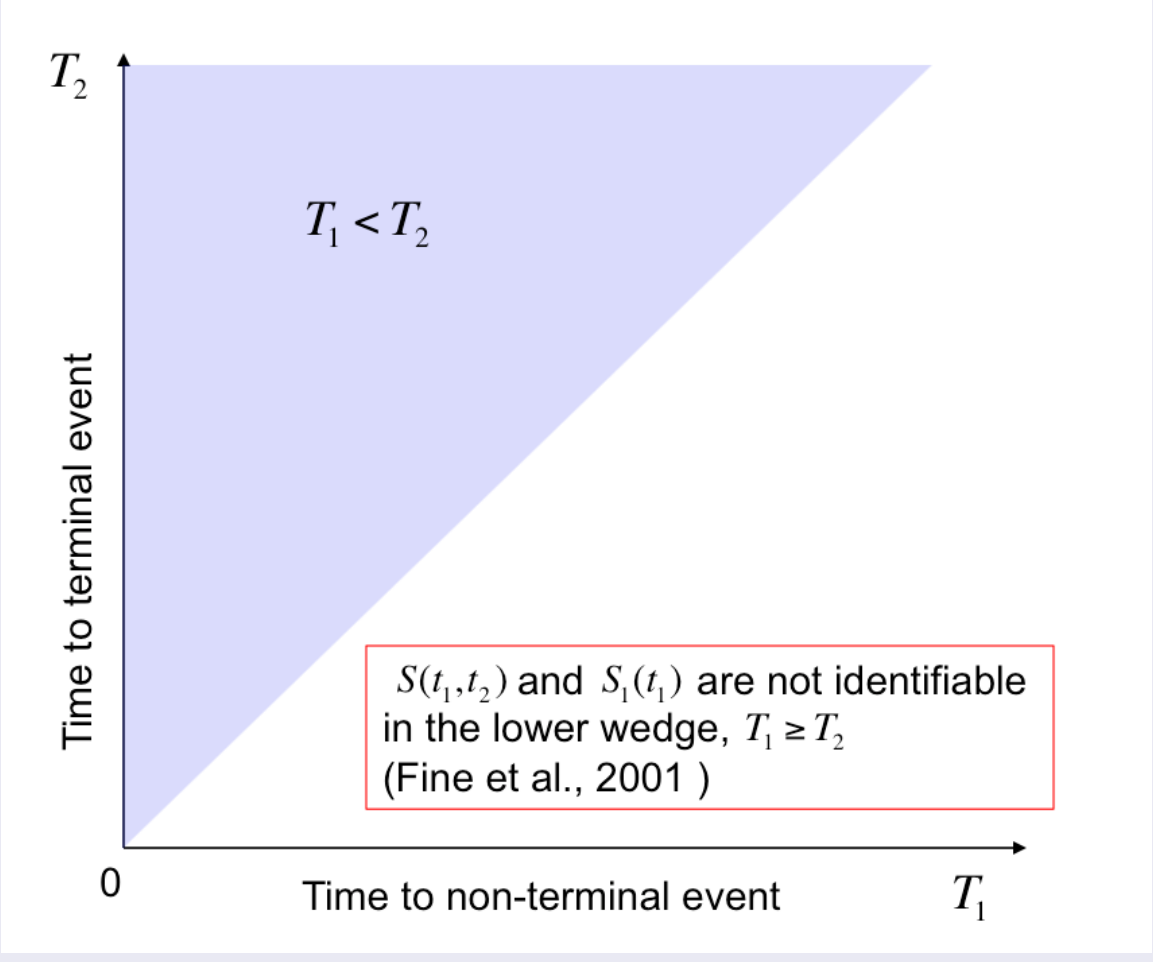
Observed events during the first 90 days		
Readmitted and subsequently died	2,254	14.0%
Readmitted and censored prior to death	2,213	13.8%
Death without readmission	7,505	46.8%
Censored prior to readmission or death	4,079	25.4%

- Primary interest lies with readmission or time-to-readmission
- We only observe readmission among folks who have not died
- Data that exhibit this structure is often referred to as *semi-competing risks* data

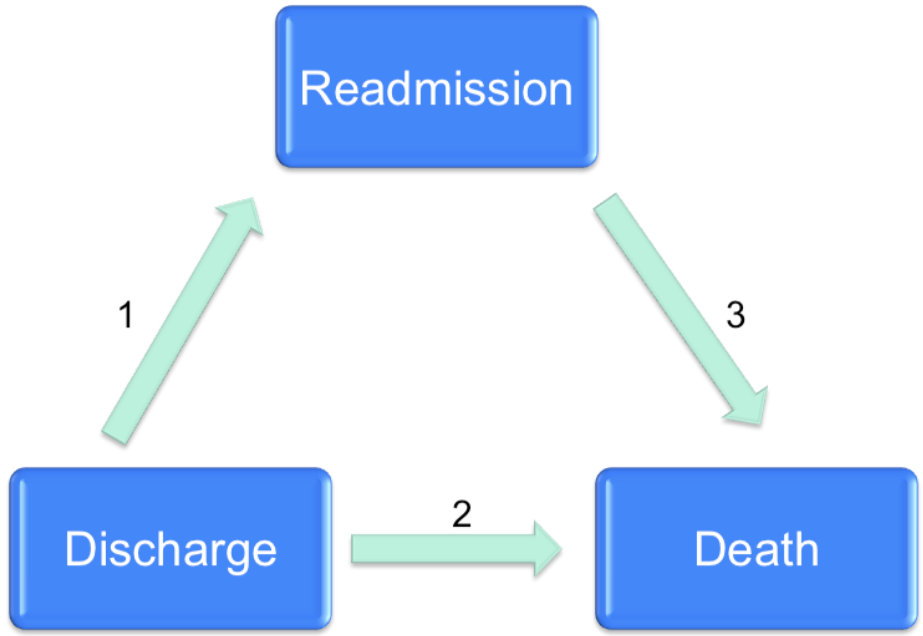
- Naïve approaches to learning about readmission might include:
  (a) logistic regression analyses
    * binary outcome
    * ignores death as a competing risk (!)
  (b) Cox regression analyses
    * time-to-readmission
    * treat death as an independent censoring mechanism (!)
  (c) composite endpoint analyses
    * first of either readmission or death
    * conflation changes the scientific question (!)
- In the profiling context, inappropriate handling of death may be problematic because:
  * a hospital may have a low readmission rates because they do a poor job of keeping patients alive
  * a hospital may have a high readmission rates because they do a good job of keeping patients alive

Semi-Competing Risks Problem

- If a patient dies prior to readmission, the time to readmission will never be observed.
- Key challenge: non-identifiability

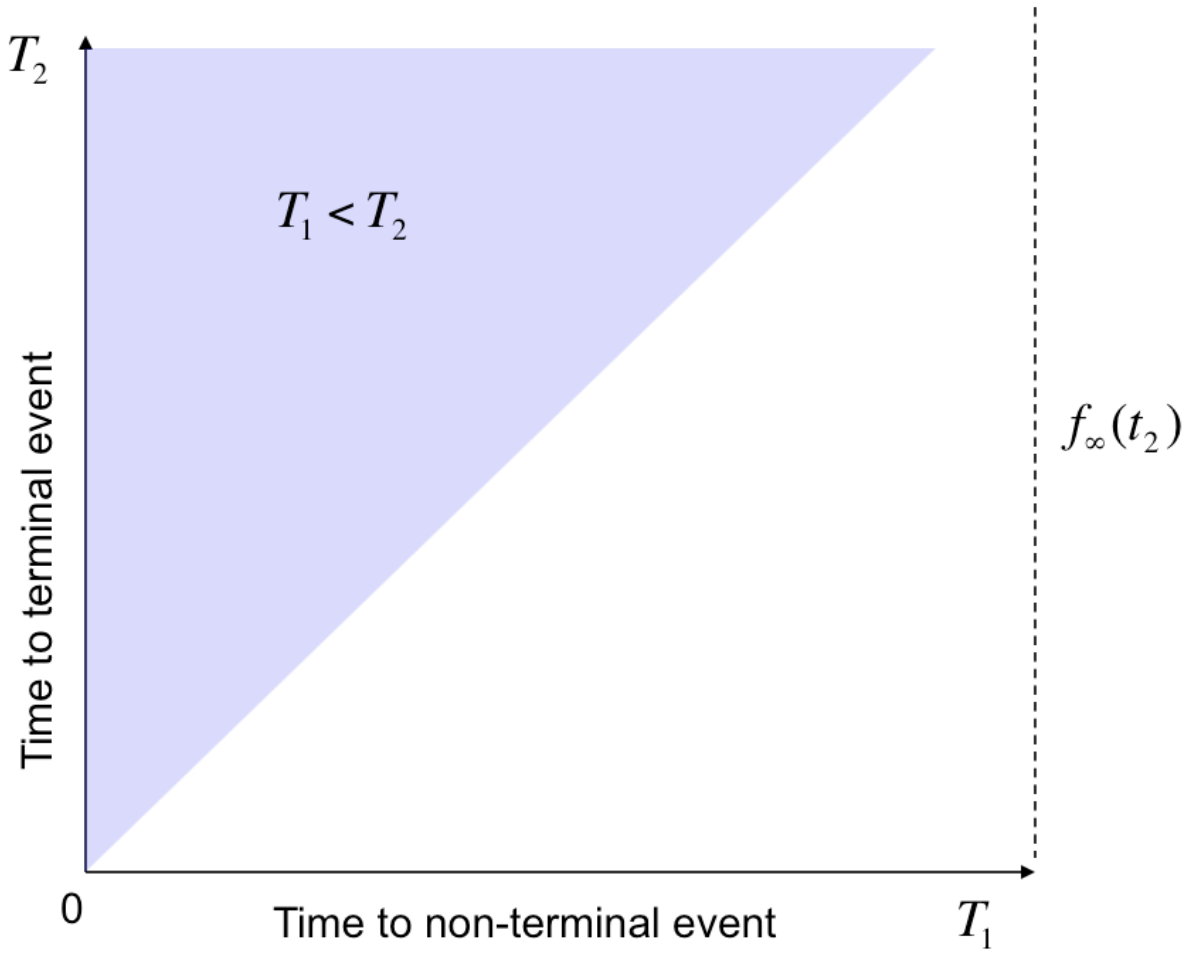


- An intuitive approach to analyzing semi-competing risks data is to view the data as arising from an underlying illness-death multi-state model.



- Movement between the states is governed by a set of transition-specific intensity or hazard functions

To permit the identifiability of the marginal density of T_1, Xu et al. (2010) set $T_1 = \infty$ if a subject experiences death prior to readmission.



Transition-Specific Hazard Functions

Modeling strategy is to place structure on the three hazard functions as follows:

$$
\begin{aligned}
h_1(t_{1i}|\gamma_i, \boldsymbol{x}_i) &= \gamma_i h_{01}(t_{1i}) e^{\boldsymbol{x}_i^\top \boldsymbol{\beta}_1}, \quad t_{1i} > 0, \\
h_2(t_{2i}|\gamma_i, \boldsymbol{x}_i) &= \gamma_i h_{02}(t_{2i}) e^{\boldsymbol{x}_i^\top \boldsymbol{\beta}_2}, \quad t_{2i} > 0, \\
h_3(t_{2i}|t_{1i}, \gamma_i, \boldsymbol{x}_i) &= \gamma_i h_{03}(t_{2i}|t_{1i}) e^{\boldsymbol{x}_i^\top \boldsymbol{\beta}_3}, \quad 0 < t_{1i} < t_{2i},
\end{aligned}
$$

* $\gamma_i \sim \text{Gamma}(\theta^{-1}, \theta^{-1})$ is a shared patient-specific frailty

- Maximization (or root solving) over a large parameter space is tricky
- Potential benefits of the Bayesian paradigm:
  * ability to incorporate substantive prior information
  * automated quantification of uncertainty
  * prediction is straightforward
  * prescriptive nature of computation
- Three main challenges:
  (1) specification of the three continuous baseline hazard functions
  (2) prior elicitation and specification
  (3) robust and efficient computational schemes

Baseline hazard functions

$$
\begin{aligned}
h_1(t_1) &= \gamma_{ji} h_{01}(t_1) \exp\left\{ \boldsymbol{X}_{ji1}^\top \boldsymbol{\beta}_1 \right\}, \quad t_1 > 0, \\
h_2(t_2) &= \gamma_{ji} h_{02}(t_2) \exp\left\{ \boldsymbol{X}_{ji2}^\top \boldsymbol{\beta}_2 \right\}, \quad t_2 > 0, \\
h_3(t_2|t_1) &= \gamma_{ji} h_{03}(t_2|t_1) \exp\left\{ \boldsymbol{X}_{ji3}^\top \boldsymbol{\beta}_3 \right\}, \quad 0 < t_1 < t_2.
\end{aligned}
$$

- One simple way forward would be to take the baseline hazard function from some parametric distribution
  - exponential/Weibull distribution
- Parametric modeling is often viewed in a negative light but it does have some advantages
  - estimation/inference tends to be (more) straightforward
  - typically more stable in data poor settings
  - prediction is more straightforward

- Nevertheless, towards a more flexible model specification, we also consider modeling each the logarithm of $h_{0g}()$ as a mixture of piecewise constant functions

$$\log(h_0(t)) = \lambda(t) = \sum_{j=1}^{J} 1_{[s_j < t \leq s_{j+1}]} \lambda_j$$

  * $\boldsymbol{s} = \{s_1, \ldots, s_J, s_{J+1}\}$ is a partition of the observed time scale

- Within the Bayesian framework we can treat J and $\boldsymbol{s}$ as 'random'
  * assign priors and update their values in the MCMC scheme
- Result is that the value of $\lambda(t)$ in any given small interval is (marginally) a mixture of piecewise constant functions
  * smooth!

The Observed Likelihood of $(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \boldsymbol{\beta}_3, \boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \boldsymbol{\lambda}_3, \boldsymbol{\gamma})$

Then the observed data likelihood for grouped or discretized survival times for n subjects has the following form in terms of the disjoint intervals:

$$\prod_{j=1}^{J_1+1} \prod_{k=1}^{J_2+1} \prod_{l=1}^{J_3+1} \exp\left\{\lambda_{1j} d_{1j} - e^{\lambda_{1j}} \sum_{m \in \mathcal{R}_{1j}} \Delta^1_{mj} \gamma_m e^{\boldsymbol{x}_m^\top \boldsymbol{\beta}_1}\right\}$$
$$\times \exp\left\{\lambda_{2k} d_{2k} - e^{\lambda_{2k}} \sum_{q \in \mathcal{R}_{1k}} \Delta^2_{qk} \gamma_q e^{\boldsymbol{x}_q^\top \boldsymbol{\beta}_2}\right\}$$
$$\times \exp\left\{\lambda_{3l} d_{3l} - e^{\lambda_{3l}} \sum_{r \in \mathcal{R}_{2l}} \Delta^{*3}_{rl} \gamma_r e^{\boldsymbol{x}_r^\top \boldsymbol{\beta}_3}\right\}$$
$$\times \prod_{m' \in \mathcal{D}_{1j}} \gamma_{m'} e^{\boldsymbol{x}_{m'}^\top \boldsymbol{\beta}_1} \prod_{q' \in \mathcal{D}_{2k}} \gamma_{q'} e^{\boldsymbol{x}_{q'}^\top \boldsymbol{\beta}_2} \prod_{r' \in \mathcal{D}_{3l}} \gamma_{r'} e^{\boldsymbol{x}_{r'}^\top \boldsymbol{\beta}_3},$$

The Prior Distributions

Our prior choices are, for $g \in \{1, 2, 3\}$:

$$
\begin{aligned}
\pi(\boldsymbol{\beta}_g) &\propto 1, \\
\boldsymbol{\lambda}_g | J_g, \mu_{\lambda_g}, \sigma^2_{\lambda_g} &\sim \mathcal{N}_{J_g+1}(\mu_{\lambda_g} \mathbf{1}, \sigma^2_{\lambda_g} \Sigma_{\lambda_g}), \\
J_g &\sim \mathcal{P}(\alpha_g), \\
\pi(\boldsymbol{s}_g | J_g) &\propto \frac{(2J_g + 1)! \prod_{j=1}^{J_g+1} (s_j - s_{j-1})}{(s_{g,J_g+1})^{(2J_g+1)}}, \\
\pi(\mu_{\lambda_g}) &\propto 1, \\
\sigma^{-2}_{\lambda_g} &\sim \mathcal{G}(a_g, b_g),
\end{aligned}
$$

and

$$
\begin{aligned}
\gamma_i | \theta &\sim \mathcal{G}(\theta^{-1},\ \theta^{-1}), \qquad i = 1, \ldots, n \\
\theta^{-1} &\sim \mathcal{G}(\psi,\ \omega).
\end{aligned}
$$

Computation and software

- MCMC via a random scan Gibbs sampling algorithm
- Most of the moves are straightforward
  * exploit conjugacies
  * Metropolis-Hastings update
- Certain moves for the baseline hazard functions require a change in the dimension of the parameter space
  * those pertaining to the number of intervals, J
  * use a reversible-jump Metropolis-Hastings-Green update
- Implemented in the `SemiCompRisks` package for `R`
  * `C` is used as the primary work engine
  * documentation includes a series of cheat sheets specific to various models that might be of interest

Application

- The data available for this study consists of information on 100% Medicare enrollees from Jan/2005 to Nov/2008.
- A total of 16,051 individuals aged 75 years or older are considered.
- For both outcomes, we (administratively) censored observation time at t = 90 days.

Objectives

- Identifying risks factors for time to readmission
- Estimating dependence between time to readmission and time to death
- Predicting probability of being readmitted

Table: Posterior medians (PM) and 95% credible intervals (CI) for hazard ratio parameters ($\exp(\boldsymbol{\beta}_g)$, $g \in \{1, 2, 3\}$) from semi-competing risks analyses based on the proposed Bayesian framework. Results are based on setting the Poisson rate parameters α_g, $g \in \{1, 2, 3\}$, to 20 for all MVN-ICAR specifications of baseline hazard functions.

		Readmission	Death prior to readmission	Death after readmission
		PM (95% CI)	PM (95% CI)	PM (95% CI)
Comorbidity index[a]	0-1	1.00	1.00	1.00
	2-3	1.03 (0.96, 1.12)	0.99 (0.93, 1.05)	0.99 (0.89, 1.10)
	≥ 4	1.26 (1.16, 1.37)	1.15 (1.07, 1.23)	1.07 (0.95, 1.21)
Race	White	1.00	1.00	1.00
	Non-white	1.27 (1.17, 1.39)	0.86 (0.79, 0.93)	1.13 (1.01, 1.28)
Gender	Female	1.00	1.00	1.00
	Male	1.10 (1.03, 1.18)	1.30 (1.23, 1.38)	1.22 (1.12, 1.34)
Age[b]		0.87 (0.84, 0.90)	1.07 (1.04, 1.10)	1.08 (1.03, 1.13)
Care after discharge	Home	1.00	1.00	1.00
	Home care	1.21 (1.12, 1.31)	1.53 (1.39, 1.69)	1.23 (1.10, 1.38)
	ICF/SNF	0.82 (0.75, 0.91)	3.46 (3.19, 3.79)	1.76 (1.54, 2.01)
	Hospice	0.18 (0.15, 0.21)	8.96 (8.25, 9.86)	3.08 (2.38, 3.99)
Hospital stay	≤ 2 weeks	1.00	1.00	1.00
	> 2 weeks	1.25 (1.12, 1.39)	1.09 (1.00, 1.20)	0.89 (0.76, 1.05)

[a] Number of diagnosis codes given during the initial hospitalization from a list of 27 disease/disorders related to prognosis following hospital discharge.

[b] Standardized so that a one-unit contrast corresponds to a difference of 5 years.

Table: Covariate profiles of the four different individuals considered for the explanatory hazard ratio and the posterior predictive distribution

	Comorbidity index	Race	Gender	Age	Care after discharge	Hospital stay
Baseline	0-1	White	Female	82	Home	≤ 2 weeks
Subject 1	≥ 4	Non-white	Male	92	Home care	> 2 weeks
Subject 2	0-1	Non-white	Female	92	Home	≤ 2 weeks
Subject 3	≥ 4	White	Male	82	Hospice	> 2 weeks

Measure of dependence between T_1 and T_2

Explanatory Hazard Ratio (EHR):

$$\frac{h_3(t_2|t_1,\gamma,\boldsymbol{x})}{h_2(t_2|\gamma,\boldsymbol{x})} = \frac{h_{03}(t_2)}{h_{02}(t_2)} \exp[\boldsymbol{x}^\top(\boldsymbol{\beta}_3 - \boldsymbol{\beta}_2)]$$

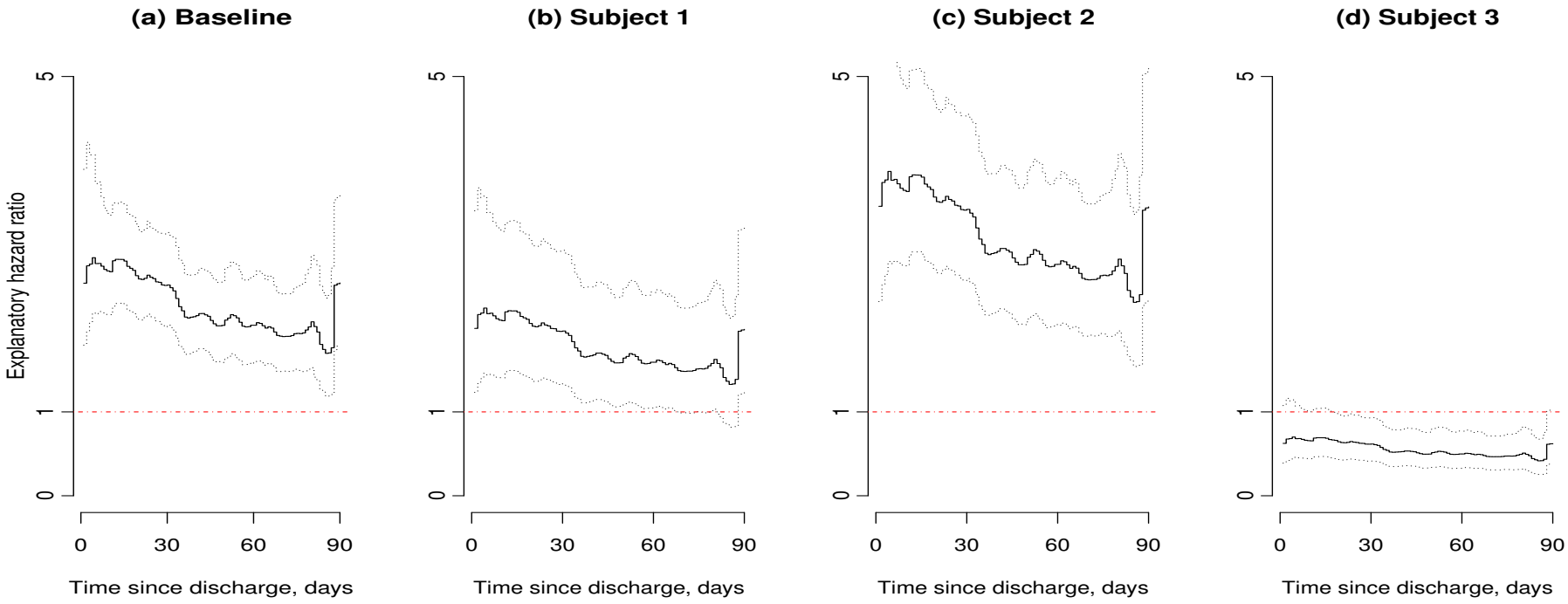


Figure: Pointwise posterior median and 95% credible intervals for the explanatory hazard ratio (EHR) for the four individuals.

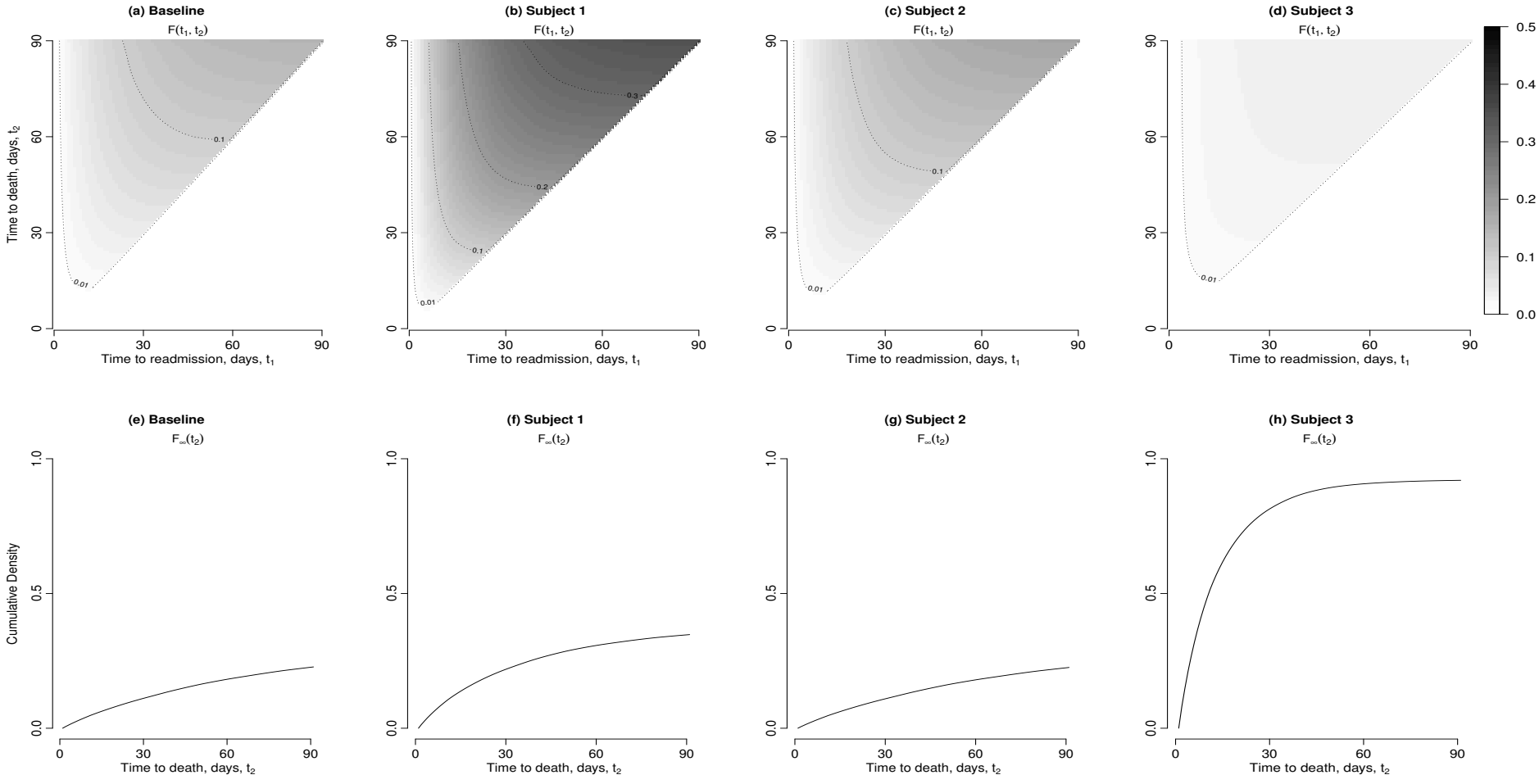


Figure: Posterior predictive distribution for four individuals; panels (a)-(d) show the posterior predictive distribution $F(t_1, t_2)$ for $t_1 \leq t_2$; panels (e)-(h) provide the posterior predictive distribution $F_\infty(t_2)$.

Final comments

- Semi-competing risks framework provides an opportunity to think about any given line of research in a different way.
  - consider the two events jointly
- Implemented in the `SemiCompRisks` package for `R`
- Cluster-correlated data (JASA, 2016), AFT model (Biometrics, 2017)

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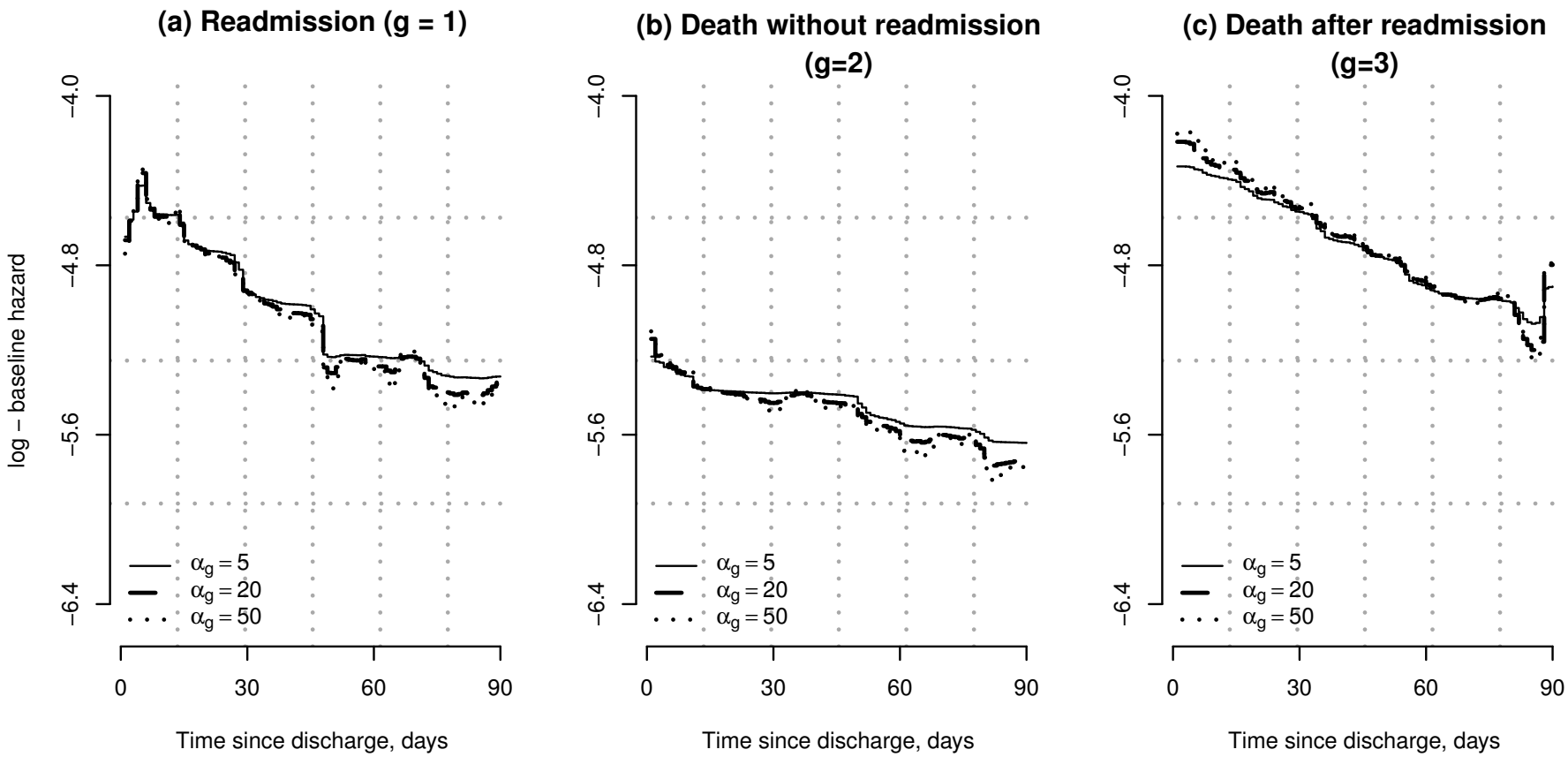

Figure: Estimates of the log-baseline hazard functions from the proposed Bayesian framework for semi-competing risks analysis. Three sets of analyses were performed, with values of α_g of 5, 20 and 50 adopted for all Poisson rate parameters.