ROYAL STATISTICAL SOCIETY
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Dear Gillian,

Response of the Royal Statistical Society's Statistics and the Law Section to letter from Forensic Science Regulator dated 27th October 2015, 'Reporting analytical results against legal limits - drug driving'

Thank you for your letter of 27th October 2015 seeking the views of the Statistics and Law Section of the Royal Statistical Society on an issue related to the comparison of analytical results to legal limits.

Our response is in three parts. The first considers your questions. The second considers wider issues that your questions raise. The third provides answers. There is also an Appendix with some numerical examples.

Part 1: Questions

You ask two questions.

- 'Might it be possible to ask the question "what is the probability of obtaining the analytical result if the sample was at the legal limit?"?'
- 'Would it reduce the level at which a sample could be reported as above the legal limit?'

Let θ_0 be the legal limit. Let θ be the true value of the blood alcohol / drug level in the person being tested. Let x be the mean of the measurements $x_1, \ldots, x_n$ where n is one of 1, 2, 3, 4, with X the corresponding random variable. Consider four possibilities for the decision that an offence has been committed.

1. The new criterion you are considering setting for blood drug analysis is to decide an offence has been committed if

$$Pr(X > x \mid X \sim N(\theta_0, \sigma^2)) < p,$$

where the notation $N(\theta_0, \sigma^2)$ denotes a Normal distribution with mean θ_0 and variance σ^2.

Here p is set at some very low level, say 0.15%, as given at the foot of page 1 of your letter. In words, this says that an offence has been committed if the measured value is such that a value as large as that would be obtained if the true value were θ_0 on only 0.15% of occasions on which the measurement was taken.

The assumptions made in this expression are:

- The distribution of the measurements X is Normal with mean θ_0.
- The standard deviation σ^2 of the mean of n measurements is $0.25\ \theta_0/(3\sqrt{n})$.

This is our interpretation of the statement 'expanded uncertainty of 25%' on page 2 in association with three standard deviations.

This criterion does not make a statement about the true blood drug level of the person tested. It makes a statement about the probability the measured value will be as high as it is, or more extreme, if the true level is at the legal limit.

2. The current criterion for blood drug analysis gives a decision that an offence has been committed if

$$Pr\ (\theta < \theta_0 \mid \theta \sim N(x,\ (0.25x/(3\sqrt{n}))^2) < p.$$

Again, p is set at some very low level, say 0.15%. In words, this says that an offence has been committed if the probability is less than p that the true value of the blood drug level of the person being tested is below the legal limit θ_0 when the mean measured value was x.

The assumptions made in this expression are:

- The distribution of the true value θ_0 is Normal with mean x.
  This represents our uncertainty about the true value as a probability distribution of a particular kind and assumes that x, the observed blood drug level, is fixed. Others will argue that θ is fixed, but unknown, and thus has no distribution attached to it.
- The standard deviation of the mean of n measurements is $0.25x/ (3\sqrt{n})$.

As above, this is our interpretation of the statement 'expanded uncertainty of 25%' on page 2 in association with three standard deviations. Again, it assumes that x, the observed blood drug level, is fixed.

3. The third possibility assumes a prior distribution for θ prior to the taking of the measurements x. Once the measurements x have been taken, a posterior distribution for θ may be obtained from which the probability that θ is less than θ_0 may be determined. However, the choice of prior distribution is problematic as there are many factors which can be considered in making the choice. Of course, these factors are open to challenge.

4. The fourth possibility is the determination of a likelihood ratio for the support given by the measurement to the proposition that the true value θ is greater than the legal limit θ_0 in contrast to the proposition that θ is less than θ_0. The evidence x is, as before, the mean of the measurements $x_1, \ldots, x_n$ where n is one of 1, 2, 3, 4. The likelihood ratio V is then

$$V = \quad f\,(x \mid \theta > \theta_0) / \quad f\,(x \mid \theta < \theta_0\,).$$

The function f is the probability density function for the distribution of $(X \mid \theta)$. One candidate for this distribution is the Normal distribution with $(X \mid \theta) \sim N(\theta, \ (0.25\,\theta/(3\sqrt{n}))^2)$ as in Possibility 1. The likelihood ratio, V, is then the ratio of two rather complicated integrals.

Three numerical examples are given in an Appendix. The first, Table 1, refers to a blood alcohol example, using Possibility 1. The second, Table 2, is a blood drug example from Possibility 1 and the third, Table 3, is a blood drug example from Possibility 2. Possibility 1 gives lower limits than Possibility 2 which is to be expected as the standard deviation is lower in the former case.

Part 2: Other comments and questions

- The standard deviation is proportional to the mean measurement. In such a situation, statisticians would often consider using logarithms of the measurements for the analysis to reduce the dependence of the standard deviation on the value of the original measurement.

- How is the standard deviation for the blood drug measurements determined?

- Would a more fundamental review of how blood alcohol and blood drug calculations are undertaken be helpful? However, this is a big task and would need resourcing to undertake.

- One of the Section committee members, Professor Jane Hutton, is in the Department of Statistics at the University of Warwick. She is willing to visit you in Birmingham to explain our ideas in person, if you think this would be helpful.

Part 3: Summary: answers to the two questions

Question 1: ***'Might it be possible to ask the question "what is the probability of obtaining the analytical result if the sample was at the legal limit?"?'***

Yes. The calculation to be made is that of

$$Pr(X > x \mid X \sim N(\theta_0, \ (0.25\,\theta_0/(3\sqrt{n}))^2) = p$$

as described in Possibility 1, where θ_0 is the legal limit, say 80 µg/L, for blood drug levels, n is the number of measurements taken and x is the observed mean of these measurements. The probability p is the probability of observing a value as large as, or larger than x and, presumably, is fixed in advance of testing in order to determine a measurement x above which an offence would have been deemed to have been committed. For given values of p and n, the corresponding value of x may be calculated. See Table 2 for examples with $p = 0.0015$ and $n = 1, 2, 3, 4$.

Question 2: ***'Would it reduce the level at which a sample could be reported as above the legal limit?'***

Yes. This approach gives lower limits as the standard deviation is lower.

I hope you have found our answers helpful. As I have mentioned above, one of the Section's committee members, Professor Jane Hutton, is willing to meet with you to explain our ideas in person, if you think this would be helpful.

Kind regards,

Prof. Colin Aitken
Chair of the Statistics and the Law Section of the Royal Statistical Society

Appendix: Numerical examples.

Blood alcohol calculations.
The standard deviation of a single measurement is taken to be 2 mg/dL, independent of the original measurement, as long as that measurement was less than 100 mg/dL, and is taken to be 2% of the original measurement if the measurement was greater than 100 mg/dL. The standard deviation of 2 mg/dL is thus fixed if the measurement is less than 100 mg/dL. The limits below are the same whether the legal limit of 80 mg/dL or the measured value is used for the basis of the calculation. The lower limits for blood alcohol levels are given in Table 1.

Table 1: Lower integer limits for blood alcohol levels assuming *n* measurements are taken with *n* = 1, 2, 3, 4 such that the probability is no greater than 0.0015 that the mean reading (of the *n* measurements) is greater than these levels if the driver's true blood alcohol level is equal to 80 mg/dL . The standard deviation is taken to be 2 mg/dL.

Number of measurements (*n*)	1	2	3	4
Lower integer limit for blood alcohol (mg/dL)	86	85	84	83

Blood drug calculations.

The standard deviation is such that 3 standard deviations equals 25% of the base reading. A comparison is wanted as to the difference in limits if the base is taken to be the legal limit of 80 *μ*g/L or to be the mean of several (*n* = 1, 2, 3, or 4) readings. If the base is taken to be the legal limit of 80 *μ*g/L, the standard deviation is taken to be one-third of 25% of 80, divided by the square root of *n*, the number of measurements of blood drug levels taken. If the base is taken to be the mean, *x* say, of *n* readings, the standard deviation is taken to be one-third of 25% of *x,* divided by the square root of *n*, the number of measurements of blood drug levels taken.

Assume first that the base figure is taken to be 80 *μ*g/L. The lower limits for the drug alcohol levels, above which it is deemed an offence, with a significance probability of 0.15% are given in Table 2. These results arise from an application of the expression in Possibility 1.

Table 2: Lower integer limits for blood drug levels assuming *n* measurements are taken with *n* = 1, 2, 3, 4 such that the probability is no greater than 0.0015 that the mean reading (of the *n* measurements) is greater than these levels if the driver's true blood drug level is equal to 80 μg/L, using the measurement 80 as the value for which the 25% is calculated.

Number of measurements (*n*)	1	2	3	4
Lower integer limit for blood alcohol (μg/dL)	100	94	92	90

Assume second that the base figure is taken to be *x* μg/L. The lower limits for the blood drug levels, above which it is deemed an offence, with a significance probability of 0.15% are given in Table 3. These results arise from an application of the expression in Possibility 2.

Table 3: Lower integer limits *x* for blood drug levels assuming *n* measurements are taken with *n* = 1, 2, 3, 4 such that the probability is no greater than 0.0015 that the mean reading (of the *n* measurements) is greater than these levels if the driver's true blood drug level is equal to 80 µg/L, using the measurement *x* as the value for which the 25% is calculated.

Number of measurements (*n*)	1	2	3	4
Lower integer limit for blood alcohol (µg/dL)	107	97	94	92

Notice the reductions in the limit from Possibility 2 to Possibility 1 are 7, 3, 2, and 2 µg/dL for *n = 1, 2, 3, 4*, respectively.