THE ROYAL STATISTICAL SOCIETY

2008 EXAMINATIONS – SOLUTIONS

ORDINARY CERTIFICATE

PAPER II

The Society provides these solutions to assist candidates preparing for the examinations in future years and for the information of any other persons using the examinations.

The solutions should NOT be seen as "model answers". Rather, they have been written out in considerable detail and are intended as learning aids.

Users of the solutions should always be aware that in many cases there are valid alternative methods. Also, in the many cases where discussion is called for, there may be other valid points that could be made.

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<u>Ordinary Certificate, Paper II, 2008. Question 1</u>

(i) $(45/60) \times 15 = 11.25$ km.

(ii) $(80/60) \times 15 = 20$ km.

(iii) $(20/30) \times 60 = 40$ minutes.

(iv) $(50/30) \times 60 = 100$ minutes = 1 hour 40 minutes.

Distance travelled $= (15 \times 0.5) + (30 \times 2) = 67.5$ km.

Time taken = 2.5 hrs.

So average speed $= 67.5/2.5 = 27$ km per hour.

Time taken $= (10/15) + (40/30) = 2$ hours.

Distance travelled = 50 km.

So average speed $= 50/2 = 25$ km per hour.

Ordinary Certificate, Paper II, 2008. Question 2

Table of tally counts

	English A	English B	English C	Total
Mathematics A	II	III	III	8
Mathematics B	IIII	II	I	7
Mathematics C	~~IIII~~ I	~~IIII~~ I	III	15
Total	12	11	7	30

Contingency table for grades of 30 students in Mathematics and English

	English A	English B	English C	Total
Mathematics A	2	3	3	8
Mathematics B	4	2	1	7
Mathematics C	6	6	3	15
Total	12	11	7	30

The modal grade in Mathematics is C.

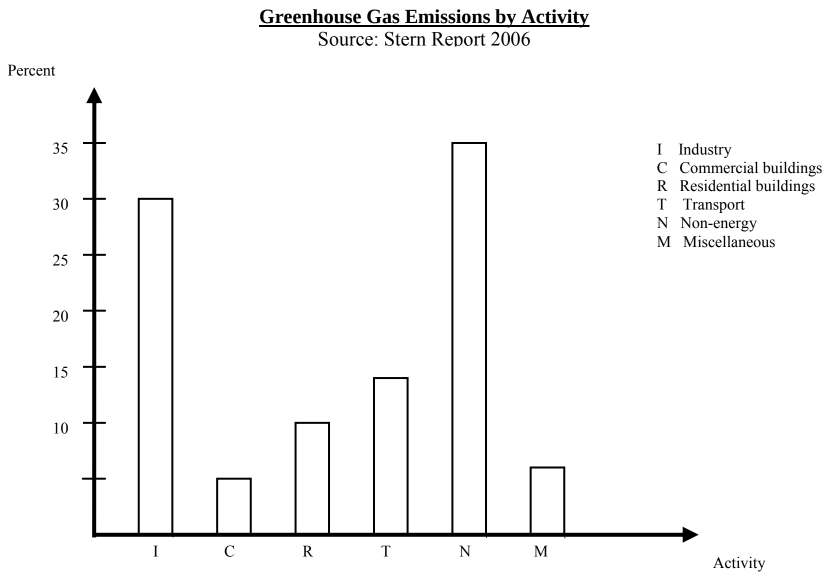
The modal grade in English is A.

Probability of a randomly selected student having As in both subjects is 2/30 or 1/15.

If Alice has a grade A in Mathematics, the probability that she has a grade A in English is 2/8 = ¼.

If David has a grade A in English, the probability that he has a grade A in Mathematics is 2/12 = 1/6.

Ordinary Certificate, Paper II, 2008. Question 3



(i) Angle for non-energy: $(35/100) \times 360 = 126^{o}$.

(ii) Angle for commercial buildings: $(5/100) \times 360 = 18^{o}$.

The overall percentage reduction would be 10% and the angles would be unchanged.

The overall percentage reduction in the second situation would be

$$\{(35+30) \times (20/100)\} + \{(5+10+14+6) \times (10/100)\} = 13 + 3.5 = 16.5\%.$$

New percentage for non-energy is $35 \times (80/100) = 28\%$.

Therefore new angle for non-energy is $(28/83.5) \times 360 = 120.7^{o}$.

New percentage for commercial buildings is $5 \times (90/100) = 4.5\%$.

Therefore new angle for commercial buildings is $(4.5/83.5) \times 360 = 19.4^{o}$.

Ordinary Certificate, Paper II, 2008. Question 4

(i) For easy puzzles, $\sum x = 36$, $n = 4$.

So mean = 36/4 = 9 minutes.

Standard deviation = $\sqrt{(\sum(x-9)^2/4)}$

$= \sqrt{\{(2^2+0^2+1^2+1^2)/4\}} = \sqrt{(6/4)} = \sqrt{1.5} = 1.2$ minutes to 1 decimal place.

(ii) For mild puzzles we have:

x	f	fx	fx^2
10	3	30	300
11	4	44	484
12	5	60	720
13	2	26	338
14	2	28	392
Total	16	188	2234

So mean = 188/16 = 11.75 minutes.

Standard deviation = $\sqrt{\{(\sum fx^2/\sum f) - (\sum fx/\sum f)^2\}}$ [or equivalent formula]

$= \sqrt{\{(2234/16) - (188/16)^2\}} = \sqrt{(139.625 - 138.0625)} = \sqrt{1.5625}$

$= 1.25$ minutes.

[Note. $n - 1$ instead of n in the denominator is acceptable, regarding this as a sample rather than a population. This gives values for the standard deviations of 1.4 and 1.3 respectively.]

Coefficient of variation = (standard deviation/mean) × 100%.

Easy: CV = 13% [could be given as 14% if $\sqrt{1.5}$ used for st dev]

Mild: CV = 11%

Difficult: CV = 12.5%

Fiendish: CV = 13%

Times taken to solve Sudoku puzzles

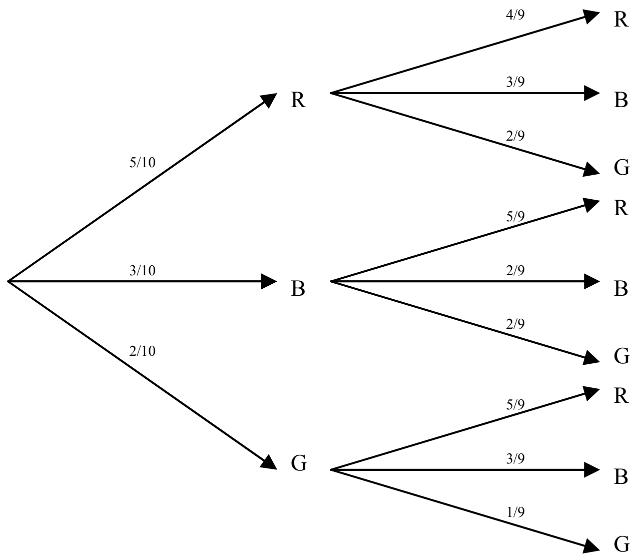
Level	Easy	Mild	Difficult	Fiendish
Mean (min)	9	11.75	18.4	25.3
Std deviation (min)	1.2	1.25	2.3	3.4
CV %	13	11	12.5	13

The mean time to solve the puzzles increases with the level of difficulty.

The variability in time also increases, as measured by the standard deviation, but the relative variability is greatest in the easy and fiendish levels.

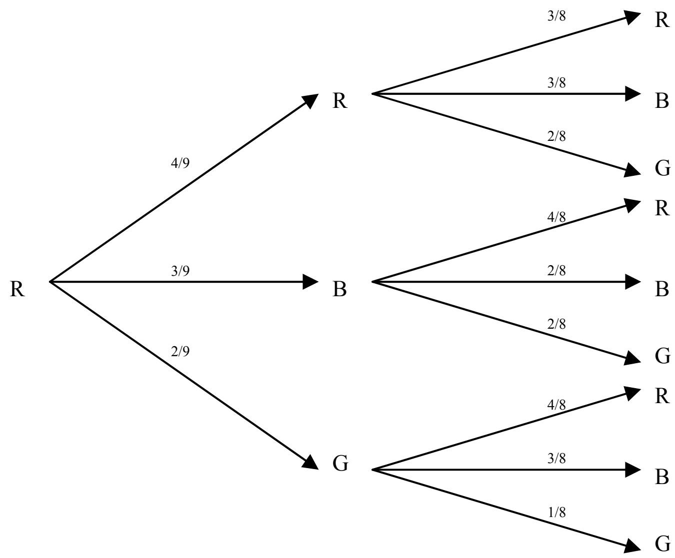
Ordinary Certificate, Paper II, 2008. Question 5

(i)



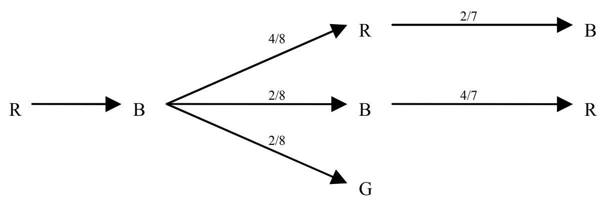
Probability = (5/10 × 4/9) + (3/10 × 2/9) + (2/10 × 1/9) = 28/90 = 14/45.

(ii)



Probability = (4/9 × 8/8) + (3/9 × 6/8) + (2/9 × 5/8) = 60/72 = 5/6.

(iii)



Probability = (4/8 × 2/7) + (2/8 × 4/7) = 16/56 = 2/7.

<u>Ordinary Certificate, Paper II, 2008. Question 6</u>

$\Sigma x = 208$ so $\bar{x} = 17.3333$.

$\Sigma y = 101$ so $\bar{y} = 8.4167$.

$\Sigma(x-\bar{x})^2 = \Sigma x^2 - (\Sigma x)^2/n$ = 4772 – (208 × 208)/12 = 1166.6667.

$\Sigma(y-\bar{y})^2 = \Sigma y^2 - (\Sigma y)^2/n$ = 1605 – (101 × 101)/12 = 754.9167.

$\Sigma(x-\bar{x})(y-\bar{y}) = \Sigma xy - (\Sigma x \Sigma y)/n$ = 2624 – (208 × 101)/12 = 873.3333.

r = 873.3333/√(1166.6667×754.9167) = 0.9306, i.e. r = 0.93 to 2 decimal places.

r is positive as higher maximum temperature is associated with higher minimum temperature; it is close to +1 indicating a high correlation.

$\hat{b} = \dfrac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2} = \dfrac{873.3333}{1166.6667} = 0.7486$, i.e. 0.75 to 2 decimal places.

$\hat{a} = \bar{y} - \hat{b}\bar{x}$ = 8.4167 – (0.7486 × 17.3333) = –4.559, i.e. –4.56 to 2 decimal places.

(i) New $\bar{x}$ = $\{1.8 \times (\text{Old } \bar{x})\} + 32$ = 63.2 (degrees Fahrenheit).

New $\bar{y}$ = $\{1.8 \times (\text{Old } \bar{y})\} + 32$ = 47.2 [47.15] (degrees Fahrenheit).

New $\Sigma(x-\bar{x})^2$ = $(1.8)^2 \times \left(\text{Old } \Sigma(x-\bar{x})^2\right)$ = 3780.00.

New $\Sigma(y-\bar{y})^2$ = $(1.8)^2 \times \left(\text{Old } \Sigma(y-\bar{y})^2\right)$ = 2445.93.

(ii) r is unchanged. $\hat{b}$ is unchanged.

New $\hat{a}$ = New $\bar{y}$ − $\left(\hat{b} \times \text{New } \bar{x}\right)$ = −0.16.

Ordinary Certificate, Paper II, 2008. Question 7

(i) (a) Trend is the basic long-term underlying movement of the series.

(b) Seasonal variation is short-term, usually regular (and in some sense seasonal), variation about the trend.

(c) A multiplicative model assumes that the components Trend, Seasonal and Irregular are multiplied together (rather than added together) to give the time series value, so that the model to explain the time series data actually observed is of the form

$$\text{Time series value} = \text{Trend} \times \text{Seasonal} \times \text{Irregular}.$$

[Cyclical variation could be included in this too.]

The chart shows a marked seasonal pattern with the highest rainfall every year in Q1 and the lowest in Q3. There appears to be a tendency for the rainfall in Q1 to be increasing with time and for that in Q3 to be decreasing with time.

Solution continued on next page

Time Series Analysis of Rainfall Data

Year/Quarter		Rainfall (mm)	4-Qtr Total(mm)	Add in pairs (mm)	Centred 4-Qtr MA (Trend) (mm)	Detrended data (to 3 dp)
2004	Q1	650				
	2	525				
			1725			
	3	125		3550	443.750	0.282
			1825			
	4	425		3600	450.000	0.944
			1775			
2005	Q1	750		3525	440.625	1.702
			1750			
	2	475		3525	440.625	1.078
			1775			
	3	100		3575	446.875	0.224
			1800			
	4	450		3575	446.875	1.007
			1775			
2006	Q1	775		3525	440.625	1.759
			1750			
	2	450		3525	440.625	1.021
			1775			
	3	75		3575	446.875	0.168
			1800			
	4	475		3575	446.875	1.063
			1775			
2007	Q1	800		3550	443.750	1.803
			1775			
	2	425		3550	443.750	0.958
			1775			
	3	75				
	4	475				

Note: 4-Qtr totals: First total $T_1 = t_1 + t_2 + t_3 + t_4$

Second total $T_2 = T_1 + (t_5 - t_1)$

Third total $T_3 = T_2 + (t_6 - t_2)$, etc

Last total, check sum of 2007 quarterly values = total obtained by difference method above.

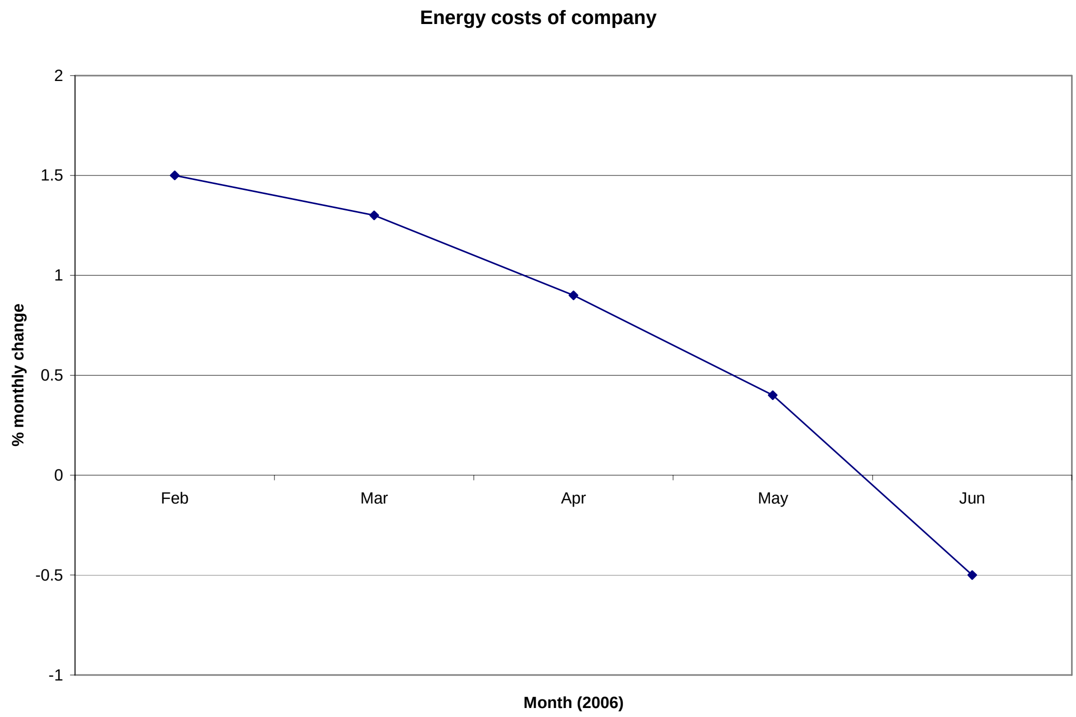
Centred 4-Qtr Moving Average values are obtained by dividing previous column by 8.

The detrended data column is Rainfall / Trend.

The trend appears to be a fairly constant value between 440 and 450 mm per quarter.

The detrended data column shows that the Q3 rainfall is markedly below the trend (28.2%, 22.4% and 16.8% of the trend in successive years), indicating that Q3 is becoming even drier than previously. By contrast the Q1 rainfall is markedly above the trend (170.2%, 175.9% and 180.3% of the trend in successive years), indicating that Q1 is becoming even wetter than previously. The rainfall in Q2 and Q4 remains much closer to the trend value throughout.

Ordinary Certificate, Paper II, 2008. Question 8



Chain-based index numbers of costs 2006

Month	Jan	Feb	Mar	Apr	May	Jun
Index	----	101.5	101.3	100.9	100.4	99.5

Fixed based index numbers of costs 2006 (January 2006 = 100)

Month	Jan	Feb	Mar	Apr	May	Jun
Index	100	101.5	102.8	103.7	104.1	103.6

Calculations

Mar	$101.5 \times 101.3\% = 102.8195 = 102.8$ to 1 decimal place
Apr	$102.8 \times 100.9\% = 103.7252 = 103.7$ to 1 d. p.
May	$103.7 \times 100.4\% = 104.1148 = 104.1$ to 1 d. p.
Jun	$104.1 \times 99.5\% = 103.5795 = 103.6$ to 1 d. p.

Comments

Costs rose every month from February to May and then dropped in June.

The rate of increase of costs was highest in February. The rate of increase of costs gradually decreased from February to May.

June was the only month showing a decrease in costs over the previous month.

The costs in May were the highest over this six-month period.

The costs in January were the lowest over this six-month period.

Overall, the costs in June were 3.6% higher than in January.