EXAMINATIONS OF THE ROYAL STATISTICAL SOCIETY

HIGHER CERTIFICATE IN STATISTICS, 2010

MODULE 2 : Probability models

Time allowed: One and a half hours

Candidates should answer **THREE** *questions.*

Each question carries 20 marks.
The number of marks allotted for each part-question is shown in brackets.

Graph paper and Official tables are provided.

Candidates may use calculators in accordance with the regulations published in the Society's "Guide to Examinations" (document Ex1).

The notation log *denotes logarithm to base* **e**.
Logarithms to any other base are explicitly identified, e.g. $\log_{10}$.

Note also that $\binom{n}{r}$ *is the same as* $^{n}C_{r}$.

This examination paper consists of 4 printed pages **each printed on one side only**.
This front cover is page 1.
Question 1 starts on page 2.

There are 4 questions altogether in the paper.

1. The random variable X has the binomial distribution with probability mass function

$$P(X = x) = \binom{2}{x} p^x (1-p)^{2-x}, \quad x = 0, 1, 2; \quad 0 < p < 1.$$

(i) Write down $E(X)$, $\mathrm{Var}(X)$ and $P(X = 2)$ in terms of the parameter p. Also find $P(X = 0 \mid X < 2)$ and $P(X = 1 \mid X < 2)$, simplifying your answers as far as possible.

(7)

(ii) Let $Y = X_1 + ... + X_{100}$ be the sum of 100 independent random variables, each distributed as X.

(a) Explain why Y has the $B(200, p)$ distribution.

(2)

(b) Use a suitable approximation to find $P(Y > 140)$ when $p = \frac{2}{3}$.

(4)

(c) Use a suitable approximation to find $P(Y > 2)$ when $p = 0.02$.

(3)

(d) Use a suitable approximation to find $P(Y \leq 197)$ when $p = 0.98$.

(4)

2. XYZ airline operates a baggage weight allowance of 25 kg per passenger. Check-in records show that the actual weight, W kg, of a randomly chosen passenger's baggage can reasonably be assumed to be Normally distributed with mean 24 and variance 1.

(i) Find $P(W > 25)$. (2)

(ii) A passenger with baggage weighing more than 25 kg is charged £5 for each kg by which the weight of his or her baggage exceeds 25 kg, all fractions of a kg being rounded up to the next whole number.

(a) If C denotes the excess baggage charge in £ for a randomly chosen passenger, find the probabilities $P(C = 0)$, $P(C = 5)$ and $P(C = 10)$. (5)

(b) Given that $P(C = 15) = 0.0013$ approximately and that $P(C > 15)$ is negligible, find $E(C)$ and $\text{Var}(C)$. (5)

(c) Assuming that 100 000 passengers independently fly with XYZ in a year, write down the mean and variance of C_T, the total excess baggage costs paid to XYZ in a year. Use a Normal approximation to find the value of C_T which is exceeded with probability 0.05. (4)

(d) Comment briefly on the assumptions made in your calculations in part (c). (4)

3. The random variable T has the exponential distribution with rate parameter λ, so that the probability density function (pdf) of T is

$$f_T(t) = \lambda e^{-\lambda t}, \quad t > 0, \quad \lambda > 0.$$

(i) Obtain the cumulative distribution function (cdf) $F_T(t)$ of T, and draw the graph of $F_T(t)$. (5)

(ii) Show that $P\left(a < T \le b\right) = e^{-\lambda a} - e^{-\lambda b}$. (2)

(iii) Given that $P(0 < T \le 1) = 2P(1 < T \le 2)$, find the value of λ to three significant figures. (6)

(iv) For any choice of c and t such that $t > c > 0$, find $P(T > t \mid T > c)$. Deduce the conditional pdf of T given that $T > c$. In a similar way, find the conditional pdf of $T - c$ given that $T > c$, and comment briefly on your results. (7)

4. Flaws in lengths of rope made by Company A occur in a Poisson process at rate λ_A per metre length, so that the number of flaws X in a length of l metres of rope has the Poisson probability mass function

$$P(X = x) = \frac{\exp(-\lambda_A l).(\lambda_A l)^x}{x!}, \quad x = 0, 1, 2, \ldots ; \quad \lambda_A > 0.$$

(i) Find the probability that there are (a) no flaws, (b) more than 2 flaws, in a 1000-metre length of rope made by company A, given that $\lambda_A = 0.002$.

(4)

(ii) Company B makes similar rope, indistinguishable in appearance from that made by Company A, in which flaws occur in a Poisson process at rate $\lambda_B =$ 0.003 per metre. A boat is rigged with 100 metres of rope from Company A and 100 metres of rope from Company B. Assuming that the lengths of rope supplied by A and B are independent, find the probability that (a) there are no flaws, (b) there is exactly one flaw, in the rigging of this boat.

(5)

(iii) (a) A manufacturer of rigging for sailing boats buys 75% of his rope from Company A and 25% from Company B. The supplier's label has become detached from a drum of rope of length 2 km which is found to have 7 flaws. Find the probability that this drum was supplied by Company A.

(6)

(b) Suppose, instead, that the rope in this drum had been found to have 8 flaws. Find the probability that this drum was supplied by Company A. Compare this probability with your answer to part (a) and comment.

(5)