EXAMINATIONS OF THE ROYAL STATISTICAL SOCIETY

HIGHER CERTIFICATE IN STATISTICS, 2015

MODULE 2 : Probability models

Time allowed: One and a half hours

Candidates should answer **THREE** *questions.*

Each question carries 20 marks.
The number of marks allotted for each part-question is shown in brackets.

Graph paper and Official tables are provided.

Candidates may use calculators in accordance with the regulations published in the Society's "Guide to Examinations" (document Ex1).

The notation log *denotes logarithm to base* ***e***.
Logarithms to any other base are explicitly identified, e.g. $\log_{10}$.

Note also that $\binom{n}{r}$ *is the same as* ${}^{n}C_{r}$.

This examination paper consists of 4 printed pages.
This front cover is page 1.
Question 1 starts on page 2.

There are 4 questions altogether in the paper.

1. (a) Let A and B be two events with $P(B) > 0$.

(i) Write down an expression for the conditional probability of A given B. (2)

(ii) Determine the conditional probability of A given B in the following cases.

(A) A and B are independent. (1)

(B) A and B are mutually exclusive. (1)

(C) $B \subset A$. (1)

(b) A diagnostic test for a disease gives a positive result with probability 0.98 for people who have the disease, and a negative result with probability 0.99 for people who do not have the disease. Suppose 3% of the population have the disease.

(i) A person is selected at random from the population and given the test. If the result is positive, what is the probability that this person has the disease? (3)

(ii) Suppose a person, initially selected at random from the population, is given the test once and the result is positive. This person is then given the test, independently, a second time and the result is again positive. What is the probability that this person has the disease? (3)

(c) Two football teams M and C each have one game left to play (not against each other) in the season. If M wins and C does not win, or if M draws and C loses, then M wins the championship. Otherwise C wins the championship. The probabilities that M wins, draws or loses the last game are $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$, respectively. The probabilities that C wins, draws or loses the last game are $\frac{2}{3}$, $\frac{1}{6}$ and $\frac{1}{6}$, respectively.

(i) What is the probability that M wins the championship? (4)

(ii) What is the probability that C has drawn the last game given that M has won the championship? (5)

2. (a) Explain carefully why a continuity correction is needed when a discrete random variable is approximated by a continuous random variable in order to calculate a probability for the discrete random variable.

(3)

(b) Explain when a Poisson approximation or a Normal approximation to a binomial probability may be used, carefully distinguishing between the two.

(4)

(c) A fair die is thrown 300 times. Find an approximation to the probability that there are fewer than 45 sixes.

(5)

(d) The number of accidents in a factory in one working week has a Poisson distribution with mean 0.2.

What is the distribution of the number of accidents in this factory in three years, comprising 150 working weeks (regarded as independent)? Find an approximation to the probability that in three years there are 35 or more accidents.

(8)

3. The random variable Y has probability density function

$$f(y) = k(y + y^3) \qquad 0 < y < 2,$$

and zero otherwise, where k is a positive constant.

(i) Show that $k = \frac{1}{6}$.

(4)

(ii) Show that the cumulative distribution function is

$$F(y) = \begin{cases} 0 & y \le 0, \\ \dfrac{y^2}{12}\left(\dfrac{y^2+2}{2}\right) & 0 < y < 2, \\ 1 & y \ge 2. \end{cases}$$

Hence find $P\left(\frac{1}{2} < Y < \frac{3}{2}\right)$.

(8)

(iii) Find the variance of Y.

(8)

4. The random variables X_i, $i = 1, \ldots, n$, are independent and are Normally distributed with means μ_i and variances σ_i^2. Their total is given by $T = \sum_{i=1}^{n} X_i$.

(i) (a) Write down $E(T)$.

(1)

(b) Write down $\text{Var}(T)$.

(1)

(c) What is the distribution of T?

(1)

(ii) A company packs parcels of books. Suppose the weights of hardback books are Normally distributed with mean 1.5 kg and standard deviation 0.5 kg and the weights of paperback books are Normally distributed with mean 0.8 kg and standard deviation 0.3 kg. Assume the weights are all independent of each other.

(a) A parcel has three hardback books and six paperback books. Find the probability that the parcel weighs more than 11 kg.

(7)

(b) Find the probability that three hardback books weigh more than six paperback books.

(5)

(c) A pallet of hardback books is to be moved by a fork-lift truck. Show that the maximum number of books which can be put on the pallet, so that there is a probability of no more than 0.01 that the combined weight of the books exceeds 100 kg, is given by the largest integer n that satisfies the inequality

$$1.5n + 1.1632\sqrt{n} - 100 < 0.$$

(5)