EXAMINATIONS OF THE ROYAL STATISTICAL SOCIETY

HIGHER CERTIFICATE IN STATISTICS, 2015

MODULE 3 : Basic statistical methods

Time allowed: One and a half hours

Candidates should answer THREE questions.

Each question carries 20 marks.
The number of marks allotted for each part-question is shown in brackets.

Graph paper and Official tables are provided.

Candidates may use calculators in accordance with the regulations published in the Society's "Guide to Examinations" (document Ex1).

The notation log denotes logarithm to base e.
Logarithms to any other base are explicitly identified, e.g. log_{10}.

Note also that \( \binom{n}{r} \) is the same as \(^rC_n\).
1. On a factory production line it is important that the time taken to assemble a component is within certain limits. Assembly times (in minutes) for a single component are recorded for a random sample of twelve factory workers and the values are as follows.

   11.7  12.8  9.9  10.6  11.6  10.6  13.1  11.2  11.6  11.9  10.9  12.7

(i) Calculate the mean and standard deviation of these observed times. 

(ii) Assuming that the underlying distribution of assembly times is Normal, calculate 95% confidence intervals for the mean and for the standard deviation of the assembly times.

(iii) In order to facilitate smooth operation of the entire production process, the assembly times must satisfy certain conditions. Test at the 5% significance level

   (a) the hypothesis that the mean assembly time in the factory is 11 minutes against the hypothesis that it is greater than 11 minutes,

   (b) the hypothesis that the standard deviation of the assembly times in the factory is 0.7 minutes against the hypothesis that it is greater than 0.7 minutes.
2. The number of births at a small maternity unit over a randomly selected three-week period is shown in the table below, classified by the day of the week on which each of the 294 births occurred.

<table>
<thead>
<tr>
<th>Day</th>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>Thu</th>
<th>Fri</th>
<th>Sat</th>
<th>Sun</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of births</td>
<td>47</td>
<td>51</td>
<td>48</td>
<td>48</td>
<td>45</td>
<td>30</td>
<td>25</td>
</tr>
</tbody>
</table>

(i) Perform a test to investigate whether the numbers of births are uniformly distributed across the seven days of the week. State your null hypothesis and conclusions clearly.

(7)

(ii) Obtain a 95% confidence interval for the proportion of births that take place at weekends (Saturday and Sunday). Use this interval to comment briefly on the suggestion that fewer births occur at weekends than on other days of the week.

(6)

(iii) A review of medical procedures is undertaken and it is desired to investigate whether, after this review, there has been a change in the proportion of births at weekends. For a three-week period randomly chosen from the six months following this review, it is observed that 68 out of 317 births are at weekends. Test the null hypothesis that the proportion of births at weekends is unchanged after the review against the alternative hypothesis that this proportion has risen.

(7)
3. An educational psychologist wishes to investigate the effect that the order of examination questions on a paper has on anxiety levels in candidates. An examination paper is prepared using identical questions in two versions. In version A the questions are presented in order of difficulty with the easiest question first, whereas in version B the questions are in reverse order with the easiest question last.

The 20 students in the class are assigned randomly to take the two different versions of the examination paper, 10 taking each version. The following are measurements of an anxiety index for the 20 students in suitable units, where low values of the index indicate lower anxiety levels.

| Version A | 24.6 | 39.3 | 16.3 | 32.8 | 28.0 | 20.6 | 21.1 | 26.7 | 24.2 | 32.9 |
| Version B | 38.6 | 34.0 | 23.6 | 30.3 | 35.9 | 22.9 | 29.5 | 39.2 | 42.9 | 33.5 |

Summary values for the above are as follows.

<table>
<thead>
<tr>
<th>Version A</th>
<th>Version B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample mean = 26.65</td>
<td>Sample mean = 33.04</td>
</tr>
<tr>
<td>Sample variance = 47.05</td>
<td>Sample variance = 43.04</td>
</tr>
</tbody>
</table>

(i) The population variances for the anxiety indices for candidates taking the two papers can be assumed to be equal. Assuming these populations to be Normally distributed, calculate a 95% confidence interval for the difference in mean anxiety levels for candidates taking the two versions of the examination. Comment briefly on what this suggests about the effect on anxiety levels of the two versions.

(10)

(ii) A statistician advises the educational psychologist that the scoring method used to produce the anxiety index measurements may not produce values which are Normally distributed. Analyse the data again, at the 5% significance level, using a two-sided Wilcoxon rank sum test. State your null and alternative hypotheses and your conclusions clearly.

(8)

(iii) Discuss the advice to use the Wilcoxon rank sum test, in particular the advantages and disadvantages of doing so.

(2)
4. In a study of supermarket checkout equipment it is found that, although checkout prices are often correct, customers can sometimes be charged more or less than the prices posted on the shelves. It is suggested that discrepancies in prices may be associated with whether or not items are on special offer compared with their normal prices. A random sample of 819 items is investigated and for each item it is noted whether the checkout equipment is undercharging, overcharging or charging the correct price. The results are shown in the contingency table below.

<table>
<thead>
<tr>
<th></th>
<th>Normal-priced items</th>
<th>Special offer items</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Undercharged</strong></td>
<td>20</td>
<td>7</td>
<td>27</td>
</tr>
<tr>
<td><strong>Overcharged</strong></td>
<td>15</td>
<td>29</td>
<td>44</td>
</tr>
<tr>
<td><strong>Correct price</strong></td>
<td>384</td>
<td>364</td>
<td>748</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>419</td>
<td>400</td>
<td>819</td>
</tr>
</tbody>
</table>

(i) Investigate whether or not price accuracy is associated with items being on special offer or not. State your null and alternative hypotheses and report your conclusions clearly.

(10)

(ii) Adapt the table to classify the normal-priced and special offer items according to whether or not they are charged correctly. Calculate a 95% confidence interval for the difference in the proportions of items which are charged incorrectly in the normal-priced and special offer categories.

(8)

(iii) Comment briefly on your answers to parts (i) and (ii).

(2)