ROYAL STATISTICAL SOCIETY
DATA | EVIDENCE | DECISIONS

# EXAMINATIONS OF THE ROYAL STATISTICAL SOCIETY

## HIGHER CERTIFICATE IN STATISTICS, 2016

## MODULE 2 : Probability models

**Time allowed: One and a half hours**

*Candidates should answer* **THREE** *questions.*

*Each question carries 20 marks.*
*The number of marks allotted for each part-question is shown in brackets.*

*Graph paper and Official tables are provided.*

*Candidates may use calculators in accordance with the regulations published in the Society's "Guide to Examinations" (document Ex1).*

*The notation* log *denotes logarithm to base* ***e***.
*Logarithms to any other base are explicitly identified, e.g.* $\log_{10}$.

*Note also that* $\binom{n}{r}$ *is the same as* ${}^nC_r$ .



This examination paper consists of 8 printed pages.
This front cover is page 1.
Question 1 starts on page 2.

There are 4 questions altogether in the paper.

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1. The Poisson random variable $X$ with parameter $\lambda > 0$ has probability mass function

$$P_X(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, \dots .$$

(i) Show that, for any integer $x \geq 0$,

$$P_X(x+1) = \frac{\lambda}{x+1} P_X(x).$$

(3)

(ii) Obtain $E(X)$ and $E(X(X-1))$. Hence show that $E(X) = \mathrm{Var}(X) = \lambda$.

(6)

(iii) Suppose that $Y$ is a Poisson random variable with mean $\mu$, that $X$ and $Y$ are independent and that $W = X + Y$. Use the relation

$$P_W(w) = \sum_{x=0}^{w} P(X = x) P(Y = w - x)$$

to show that $W$ is a Poisson random variable with mean $\lambda + \mu$.

(4)

(iv) A manufacturer has two conveyor belts, one of type $C$ and the other of type $D$. The numbers of breakdowns per day on these belts, $X$ and $Y$, are independent Poisson random variables with means 1.5 and 0.5 for belts $C$ and $D$ respectively.

(a) Find the conditional probability that if there is exactly one breakdown on a given day on either belt $C$ or belt $D$, but not both, then it is conveyor belt $C$ that fails.

(4)

(b) What is the probability that the factory will experience at most one breakdown in a 5-day period?

(3)

2. A sequence of independent Bernoulli trials with probability of success $p$ is performed. Let the random variable $X$ be the number of failures before the first success.

(i) Find the probability mass function of $X$, confirming that the sum over all possible values of $X$ is one.

(5)

(ii) Obtain $E(X)$.

(4)

(iii) The probability of an enemy aircraft penetrating friendly airspace is 0.01.

(a) What is the probability that the first penetration of friendly airspace is accomplished by the 80th aircraft to attempt the penetration, assuming penetration attempts are independent?

(2)

(b) What is the probability that it will take more than 80 attempts to penetrate friendly airspace?

(5)

(iv) Consider $Y$, the total number of failures before the second success. Find the probability mass function of $Y$. By considering the mean of $X$, show that

$$E(Y) = 2\left(\frac{1-p}{p}\right).$$

(4)

3. (a) Large and small bottles of water are manufactured in a factory. The volume of water in the large bottles follows a Normal distribution with mean 1.5 litres and standard deviation 0.01. The volume of water in the small bottles follows an independent Normal distribution with mean 0.5 litres and standard deviation 0.008.

(i) One large bottle and three small bottles are chosen at random. What is the probability that the volume in the large bottle is greater than the total volume in the three small bottles?

(3)

(ii) Find the distribution of the total volume of 10 large bottles and 3 small bottles.

(6)

(b) A manufacturer produces bags of sweets. The weights of the bags are independently distributed with mean 30 g and standard deviation 5 g. A random sample of 50 bags of sweets is taken.

(i) State the approximate distribution of the mean weight of the bags in the sample, quoting any results you use.

(3)

(ii) Use this distribution to find the probability that the mean weight of the bags in the sample is less than 29 g.

(3)

(iii) Suppose instead a random sample of size $n$ bags is taken. It is desired that the probability that the mean weight of bags in the sample is below 29 g is less than 0.05. Find the minimum required value of $n$.

(5)

4. (a) The continuous random variable $X$ has probability density function

$$f(x) = \alpha(1-x)^{\alpha-1}, \quad 0 < x < 1, \ \alpha > 0.$$

(i) Find the cumulative distribution function, $F(x)$, of $X$.
(3)

(ii) Find $P(0.25 < X < 0.75)$.
(4)

(iii) Use $F(x)$ to obtain the median of $X$.
(3)

(b) The continuous random variable $Y$ has probability density function given by

$$f(y) = 9ye^{-3y}, \quad y \geq 0.$$

(i) Obtain $E(Y)$.
(6)

(ii) What is $P(Y < 3)$?
(4)

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