THE ROYAL STATISTICAL SOCIETY

2015 EXAMINATIONS − SOLUTIONS

HIGHER CERTIFICATE – MODULE 3

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The solutions are intended as learning aids and should not be seen as "model answers".

Users of the solutions should always be aware that in many cases there are valid alternative methods. Also, in the many cases where discussion is called for, there may be other valid points that could be made.

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1.

(i) $\bar{x} = \frac{\sum x}{n} = \frac{138.6}{12} = 11.55 \text{ cm.}$ 1 (11.55)

$$s^2 = \frac{1}{n-1}\left\{\sum x^2 - \frac{\left(\sum x\right)^2}{n}\right\} = \frac{1}{11}\left\{1611.34 - \frac{138.6^2}{12}\right\} = 0.955$$

So s = 0.9775 cm. 1 (0.9775)

(Accept both answers if direct from calculator)

TOTAL 2

(ii) 95% C.I. for μ is given by $\bar{x} \pm t.\frac{s}{\sqrt{n}}$ (t on 11 d.f.) 1 (method)

1 (11 d.f. EITHER here OR in (iii))

which is $11.55 \pm 2.201 \times \frac{0.9775}{\sqrt{12}}$ i.e. **(10.929, 12.171)** 1 (2.201)

1(10.929), 1(12.171)

95% C.I. for σ^2 given by $\left(\frac{(n-1)s^2}{\chi^2_{.025}}, \frac{(n-1)s^2}{\chi^2_{.975}}\right)$ (χ^2 values on 11 d.f.).1 (method)

which is $\left(\frac{11 \times 0.955}{21.920}, \frac{11 \times 0.955}{3.816}\right)$ i.e. (0.479, 2.75). 1 (21.920), 1 (3.816)

Hence 95% C.I. for σ is **(0.692, 1.660).** 1 (0.692), 1 (1.660)

TOTAL 10

(iii) (a) $H_0 : \mu = 11$ vs $H_1 : \mu > 11$.

Test statistic t = $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{11.55 - 11}{0.9775/\sqrt{12}} = 1.949.$

1 (method), 1 (1.949)

Compare with upper 5% point of t_{11} which is 1.796. 1 (1.796)

Test statistic is significant, so reject H_0

Implies mean assembly time is significantly longer than required.

1 (correct conclusion)

(b) $H_0 : \sigma^2 = 0.7^2$ vs $H_1 : \sigma^2 > 0.7^2$

Test statistic is $\frac{(n-1)s^2}{\sigma_0^2} = \frac{11 \times (0.9775)^2}{0.7^2} = 21.450.$

1 (method), 1(21.450)

Compare with upper 5% point of χ^2_{11} which is 19.675. 1 (19.675)

Test statistic is highly significant, so reject H_0.

Implies assembly times are too variable. 1 (correct conclusion)

TOTAL 8

2. (i) Test H_0: distribution of deliveries is uniform against 1(null hyp correct)

H_1: distribution is not uniform.

Total number of deliveries is 294, so $E_i = 42$ for each day. 1 (E = 42)

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = \frac{17^2 + 5^2 + 9^2 + 6^2 + 6^2 + 3^2 + 12^2}{42} = \frac{620}{42} = 14.76.$$ 1 (method)

1(14.76)

5% point of χ^2 distribution on 6 degrees of freedom is 12.592. 1 (6 df), 1 (12.592)

So our result is significant, we reject the null hypothesis and conclude that the distribution of deliveries does not appear to be uniform. 1 (conclusion correct)

TOTAL 7

(ii) $\hat{p} = \frac{25+30}{294} = 0.1871$. $s.e.(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.1871 \times 0.8129}{294}} = 0.02274$. So 95% confidence interval is $0.1871 \pm 1.96 \times 0.02274$ 1 (0.1871 or 55/294), 1 (1.96)

1 (CI formula)

i.e. (0.1425, 0.2317). 1(0.1425), 1 (0.2317)

If the distribution was uniform, then we would expect a proportion $\frac{2}{7} = 0.2857$ of deliveries to be made at weekends. Our confidence interval lies well below this value, indicating that fewer deliveries than expected are made at weekends. 1 (comment)

TOTAL 6

(iii) We have $\hat{p}_1 = 0.1871$ as above and $\hat{p}_2 = \frac{68}{317} = 0.2145$. 1 (0.2145 or 68/317)

Wish to test $H_0: p_1 = p_2$ against $H_1: p_1 < p_2$.

Pooled sample proportion is $\hat{p} = \frac{55+68}{294+317} = 0.2013$ and 1 (0.2013 or 123/611)

$$s.e.(p_1 - p_2) = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \sqrt{0.2013 \times 0.7987 \times \left(\frac{1}{294} + \frac{1}{317}\right)} = 0.0325.$$

1 (method), 1 (0.0325)

Then Z = $\frac{\hat{p}_1 - \hat{p}_2}{s.e.(\hat{p}_1 - \hat{p}_2)} = \frac{-0.0274}{0.0325} = -0.843.$ 1 (-0.843)

Comparing with the lower 5% point of N(0, 1) which is -1.645, our value is not significant so we do not reject the null hypothesis. Thus we have no evidence that the proportion of births at weekends has risen significantly. 1 (c.f. -1.645)

1 (conclusion correct)

TOTAL 7

3(i) Assuming equal population variances, the confidence interval for $\mu_A - \mu_B$ is given by

$$(\bar{x}_A - \bar{x}_B) \pm t.s\sqrt{\frac{1}{n_A} + \frac{1}{n_B}}$$ where s is the pooled sample variance. 1 (method)

We have $s^2 = \frac{s_A^2 + s_B^2}{2}$ (equal sample sizes) = $45.045 \Rightarrow s = 6.712$. 1 (method)

1 (6.712 or 45.045)

From tables, the t value on 18 degrees of freedom is 2.101. So we have interval 1 (18 df)

1 (2.101)

$$(26.65 - 33.04) \pm 2.101 \times 6.712 \times \sqrt{\frac{1}{10} + \frac{1}{10}} = -6.39 \pm 6.307$$ 1 (method)

i.e. (-12.697, -0.083). 1 (-12.697), 1 (-0.083)

(0.083 to 12.697 also acceptable)

The CI does not include zero, indicating that the mean anxiety levels produced by the two versions may differ. Version A, with the easy questions first, appears to produce lower levels of anxiety. 1 (means differ), 1 (lower levels in A)

TOTAL 10

(ii) H_0 : the two samples come from the same distribution (equal medians). 1 (null hyp)

H_1 : the two samples come from different distributions. 1 (alt hyp)

The ranks are: 1 (correct test attempted)

Version A

Value	24.6	39.3	16.3	32.8	28.0	20.6	21.1	26.7	24.2	32.9
Rank	7	19	1	12	9	2	3	8	6	13

Version B

Value	38.6	34.0	23.6	30.3	35.9	22.9	29.5	39.2	42.9	33.5
Rank	17	15	5	11	16	4	10	18	20	14

1 (correct ranks)

The sums of the ranks are 80 (Version A) and 130 (Version B). 1 (80), 1 (130)

The lower of these two values is 80. 1 (choose lower)

We compare this with the value from tables with sample sizes 10 and 10 which is 78. 1 (78)

Since our value is higher than this critical value we do not reject the null hypothesis.

1 (do not reject null)

i.e. the median anxiety levels for the two versions do not appear to be significantly different

1 (conclusion)

TOTAL 10

4(i) H_0 : there is no association between charging discrepancies and normal/special offer.

H_1 : there is an association. 1 (null hyp), 1(alternative hyp)

For each table cell, expected frequency = $\frac{\text{row total} \times \text{column total}}{\text{grand total}}$. These are given in brackets below: 1 (method or implied by values below)

20 (13.81)	7 (13.19)	27
15 (22.51)	29 (21.49)	44
384 (382.68)	364 (365.32)	748
419	400	819

1 (all E's correct)

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 6.19^2\left[\frac{1}{13.81} + \frac{1}{13.19}\right] + 7.51^2\left[\frac{1}{22.51} + \frac{1}{21.49}\right] + 1.32^2\left[\frac{1}{382.68} + \frac{1}{365.32}\right]$$

= 5.679 + 5.130 + 0.009 = 10.818. 1 (method), 1(10.818)

Comparing with tables and using 2 degrees of freedom, the critical 5% point is 5.991. The calculated value is thus highly significant. So we have strong evidence to reject the null hypothesis in favour of the alternative hypothesis. i.e. the incidence of discrepancy in checkout and shelf prices is highly associated with whether or not items are on special offer.

1 (2 df), 1 (5.991), 1 (reject null hyp)

1 (correct conclusion)

TOTAL 10

(ii) Now have

	Normal	Special offer
Incorrect price	35	36
Correct price	384	364
Total	419	400

1 (table adaptation)

$\hat{p}_1 - \hat{p}_2 = \frac{35}{419} - \frac{36}{400} = 0.0835 - 0.09 = -0.0065.$ 1 (0.0835 or 35/419), 1 (0.09 or 36/400)

Estimated s.e. of this difference is $\sqrt{\frac{0.0835 \times 0.9165}{419} + \frac{0.09 \times 0.91}{400}} = 0.0197$ 1 (method)

So the 95% confidence interval is $-0.0065 \pm 1.96 \times 0.0197$ which is (-0.045, 0.032).

1 (CI method), 1 (1.96), 1 (-0.045), 1 (0.032)

TOTAL 8

(iii) In (i) we found a difference in price accuracy between the normal priced and special offer items. However, the interval in (ii) contains zero, suggesting that there is no difference in the proportions of incorrect prices for the two categories. This is because the main discrepancy (i.e. largest contributions to overall χ^2) in (i) was between the overcharged/undercharged items whereas in (ii) these categories have been amalgamated.

1 ('appear inconsistent')

1 (valid explanation)

TOTAL 2