EXAMINATIONS OF THE ROYAL STATISTICAL SOCIETY

HIGHER CERTIFICATE IN STATISTICS, 2017

MODULE 2 : Probability models

Time allowed: One and a half hours

Candidates should answer **THREE** *questions.*

Each question carries 20 marks.
The number of marks allotted for each part-question is shown in brackets.

Graph paper and Official tables are provided.

Candidates may use calculators in accordance with the regulations published in the Society's "Guide to Examinations" (document Ex1).

The notation log *denotes logarithm to base* ***e***.
Logarithms to any other base are explicitly identified, e.g. $\log_{10}$.

Note also that $\binom{n}{r}$ *is the same as* ${}^{n}C_{r}$.

This examination paper consists of 8 printed pages.
This front cover is page 1.
Question 1 starts on page 2.

There are 4 questions altogether in the paper.

1. The random variable X follows the binomial distribution with probability mass function

$$P_X(x) = \binom{3}{x} p^x (1-p)^{3-x}, \quad x = 0, 1, 2, 3, \quad 0 < p < 1.$$

(i) Write down $E(X)$, $\text{Var}(X)$ and $P(X = 1)$ in terms of the parameter p. Also find $P(X = 0 \mid X < 2)$ and $P(X = 1 \mid X < 2)$, simplifying your answers as far as possible. (7)

(ii) Let $Y = X_1 + X_2 + \ldots + X_{100}$ be the sum of 100 independent random variables, each distributed as X.

(a) Explain why Y has the $B(300, p)$ distribution. (2)

(b) Use a suitable approximation to find $P(Y > 220)$ when $p = 0.7$. (4)

(c) Use a suitable approximation to find $P(Y > 5)$ when $p = 0.02$. (3)

(d) Use a suitable approximation to find $P(Y \leq 293)$ when $p = 0.98$. (4)

2. In a certain large population, the heights X of adult females can be assumed to have a Normal distribution with mean 160 cm and variance 100 cm^2.

(i) (a) Find the probability that a randomly chosen woman is more than 174 cm tall.

(3)

(b) Find the probability that a randomly chosen woman is within 4 cm of the population mean height.

(3)

(c) Given that a randomly chosen woman is more than 174 cm tall, find the probability that she is more than 179.6 cm tall.

(3)

(ii) A sample of 3 women is chosen at random from this population.

(a) Find the probability that all three women in the sample are within 4 cm of the population mean height.

(2)

(b) Find the probability that the mean height of the sample is within 4 cm of the population mean height.

(3)

(iii) The heights Y of the adult males in this large population can be assumed to have a Normal distribution with mean 180 cm and variance 100 cm^2. Married couples of a man and a woman have heights which are correlated with $\rho = 0.5$.

(a) Find the distribution of the difference in height $Y - X$ of a married couple.

(3)

(b) Find the probability that the height of a wife is more than that of her husband.

(3)

3. (i) The continuous random variable X has the exponential distribution with probability density function $f(x)$ given by

$$f(x) = \lambda \exp(-\lambda x) \qquad x > 0,\ \lambda > 0.$$

Derive the cumulative distribution function (cdf) $F(x)$ of X. Sketch the cdf. In the case when $\lambda = \frac{1}{3}$, use $F(x)$ to find the median of X, and indicate it on your sketch.

(9)

(ii) Show that $E(X) = \dfrac{1}{\lambda}$.

(3)

(iii) For any a, $b > 0$ find $P(X > a + b \mid X > a)$. Comment on the result in regard to the use of the exponential distribution as a lifetime distribution.

(4)

(iv) $X_1, X_2, ..., X_n$ are independent and identically distributed exponential variables, each with the same distribution as X. State the distribution of $Y = \sum_{i=1}^{n} X_i$ and its mean and variance.

(4)

4. (i) In Newtopia the weather on any day is sunny with probability 0.8 and snowy with probability 0.2. The weather on any particular day is independent of the weather on any different day.

(a) Find the probability that the weather is the same for the next four days. (3)

(b) Find the probability that exactly two of the next three days are snowy. (3)

(c) In Newtopia the probability that a train is late is 0.1 when it is sunny, but 0.6 when it is snowy. If the train arrived late yesterday, what is the probability that it was sunny yesterday? (4)

(ii) In Tapland it is also sunny or snowy each day in winter. The weather on different days is not independent, with

P(next day is sunny | today is sunny) = 0.8,

P(next day is snowy | today is snowy) = 0.3.

(a) Given that today is snowy, what is the probability that the next three days are snowy? (2)

(b) Given that today is sunny, what is the probability that exactly two of the next three days are snowy? (4)

(c) Let p denote the probability that any day is snowy. Find the value of p. (4)

BLANK PAGE

BLANK PAGE

BLANK PAGE