## SECTIONS AND LOCAL GROUP MEETING REPORTS.

MOST POWERFUL INFERENCE AFTER MODEL SELECTION.

Written by **Gilbert MacKenzie** on February 20th. 2022. Posted in Section and local group meeting reports April 7th, 2023

The Northern Ireland local group of the RSS held an online meeting on Wednesday, February 4th, at 1pm (GMT), using MS Teams.

The speaker was Dr. Gerda Claeskens, The Catholic University of Leuven, Belgium, EU.

Gerda's main message was simply when confidence intervals or hypothesis tests are constructed for parameters in a selected model, most likely misspecified, the classical formulas no longer apply in the sense that computed *p*-values computed via classical formulas are too small and confidence intervals are too narrow. This is a serious problem which need to be addressed.

The main starting point of selective inference (PoSI) is to condition on the event of having selected the model that one wishes to use for inference. As a tool for valid inference, she used *confidence distributions*, conditional on the event of having selected a model.

The effect of post-selection inference, is to increase the conventional *Frequen*tist confidence coefficient at the  $(1-\alpha)\%$  level (e.g., 1.96, when  $\alpha=0.05$ ) and the result is termed the PoSI constant and denoted K. Since  $\alpha \in (0, 1)$  we can plot in 2-D the confidence interval end points for different  $\alpha s$  (Y-axis) against the parameter values (X-axis). This useful plot is known as a confidence curve and can be used to compare confidence distributions (see Figure). Notice that the



Figure: (a) LHS shows, end-points  $\hat{\beta}_j \pm 1.96 \operatorname{se}(\hat{\beta}_j)$  plotted, näive 95%CI=[0.68, 3.53] (a covariate from Levee data) and (b) RHS compares curves näive (red), PoSI (blue) and PoSI Optimal (black) based on different confidence distributions.

RHS panel allows a comparison in terms of coverage and that the optimal PoSI curve improves on what Gerda called näive PoSI. She explained that optimal distributions can be based on the CDF s of Pivots, or in the sufficient statistics

framework, by conditioning on the sufficient statistics for nuisance parameters. In addition, one also conditions on the method of selection, e.g. (a) significance hunting, (b) all subset selection by BIC, forward selection by AIC and others.

Next she went through an AIC example (with m = 3 candidate models) in detail showing that each possible model selection in the candidate set  $\mathcal{M}$ corresponded to a partition of the sample space defined by the test statistic. Each model selection method has this property. Thus the observation vector  $Y_n = \bigcup_{j=1}^m A_j$  where  $A_j$  is the selection region for model j.

With this machinery in place Gerda established optimal PoSI confidence distributions for the Normal linear model saying this the method could be applied to the following selection methods: AIC, BIC, Cross-validation, Lasso, Lars (polyhedral regions), and Significance 'hunting' with likelihood ratio tests, t-tests, F-tests, etc. She extended her results to the Exponential family (including discrete cases) and to Generalized Linear Models with a canonical link function.

All in all, this talk was a *tour de force* of PoSI methodology. It was received with acclaim by a very attentive and appreciative audience.

A short discussion ensued which dealt with the consequences of failing to make appropriate adjustments for selection in individual studies and more generally how this impacted on Science and the 'so-called' reproducibility crisis. The audience was left with considerable food for thought.

Afterwards, the Chair thanked the speaker on a very stimulating talk and concluded the meeting by thanking everyone for their attendance and support.

## Key References

Ali Charkhi, Gerda Claeskens (2018). Asymptotic post-selection inference for the Akaike information criterion. Biometrika, Volume 105, Issue 3, Pages 645–664, https://doi.org/10.1093/biomet/asy018.

Andrea C. Garcia-Angulo and Gerda Claeskens (2022). Exact uniformly most powerful post-selection confidence distributions. Scandinavian Journal of Statistics. https: //doi.org/10.1111/sjos.12581 (March 2022).